Noise and Fault Diagnosis Using Control Theory

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Abstract : The aim of this paper is to describe an advanced method of the fault diagnosis using Control Theory with reference to a crack detection, a new way to localize the crack position under influence of the plant disturbance and white measurement noise on a rotating shaft. As the first step, the shaft is physically modelled with a finite element method as usual and the dynamic mathematical model is derived from it using the Hamilton - principle and in this way the system is modelled by various subsystems. The equations of motions with a crack are established by the adaption of the local stiffness change through breathing and gaping[1] from the crack to the equation of motion with an undamaged shaft. This is supposed to be regarded as a reference systrm for the given system. Based on the fictitious model of the time behaviour induced from vibration phenomena measured at the bearings, a nonlinear state observer is designed in order to detect the crack on the shaft. This is the elementary NL-observer(EOB). Using the elementary observer, an Estimator(Observer Bank) is established and arranged at the certain position on the shaft. In case, a crack is found and its position is known, the procedure for the estimation of the depth is going to begin.

Keywords : dynamic behaviour, nl-observer, estimator, crack detection, crack position, crack depth, noise

I. Introduction

As a classical method, there are some ways to find the split on the shaft. For example to analyse the vibration peaks, acoustics, to measure the oil temperature by the costdown and by the transition of the resonance[2]. To use these methods the expert on these subject are needed. Nevertheless, they miss finding out the clues of a crack very often and from time to time the crack symptom are misunderstand as an effect from a damaged bearing. Except the analysis method, there are some other methods: namely, to compare the time signal between damaged stage in the operation and undamaged stage in the initial stage[3], to look for the sensibility of eigen value[4] and to use modal observer under model reduction. A similar way to detect a crack is given by [5]. They have contributed in a way to a crack detection.

But as a physical model they have used mass-lump model. It means that the results may be not do exact in terms of the model reduction and physical modelling. These methods are hardly to offer clear relationships between phenomena and change of the stiffness which are necessary for the localization of the crack on the shaft. Therefor, a new method based on the theory of disturbance rejection control[6,7] is suggested for the detection of the crack, estimating the position with respect to constant crack depth. As an indicator for the existence of a crack, the nonlinear dynamic effects, appeared by the change of the stiffness coefficients due to the rotation of the cracked shaft, are going to be investigated. These effects related to the measurements on the bearings, are one of the important clues to determinate the existence of the crack on the rotating shaft. But it is very difficult to set up the clear relation between crack and caused phenomena in the time domain operation. This is the main task in the area of the crack problem too. But in case of apperance of noise on the system the results of the process may be able to be corrupted by noise and lead to the false detection, localization. Therefore, it is necessary to look for the effects of noise on the system.

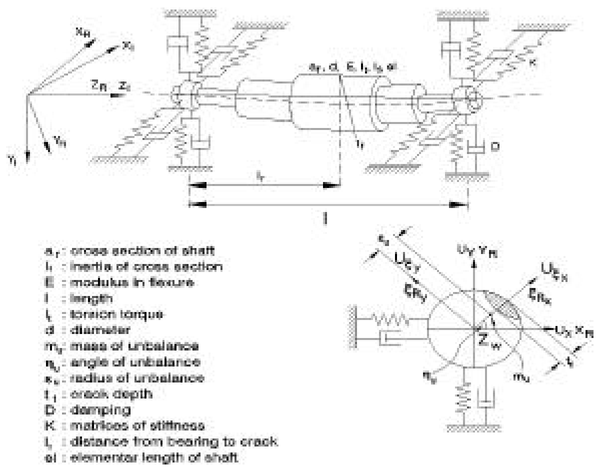

Fig. 1. Physical model of the rotor

First of all, the basic state observer is established in the way to modify the given system into the extended system with a linear fictitious model for the nonlinear system behaviour. In this consideration the effects of the extended system which may be nonlinear, are interpreted as an internal or external disturbance which is unknown at the initial stage.

The unknown nonlinear effects are going to be approximated by the additional time signals yielded by elementary state observer. Because of using FEM model, it is not necessary to calculate the relative compliance of the crack. Normally the elementary stiffness matrix for an undamaged rotor is given in the stage of the construction and the stiffness corresponding to the crack is able to be calculated[8,9,10].

As an example the physical model, which is divided into N(=7) finite sub-shafts is modelled[11,12]. Every one is called a subsystem. At both ends of the shaft there exist dynamics of the bearings. They have the task of system control. For the initial data needed in the operating system the displacements of jounals are measured up on the bearings at the left and right side of the shaft. It is assumed

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that the material properties are homogenous. The geometrical data and other detailed information are given in the appendix.

II. Equation of motion

Assuming that there is only small deviation from motion and no redundant coordinate[13,14], the system including 3 harmonic unbalances in the 3rd, 4th and 5th subsystems in the middle of the shaft

$$M_g \ddot{q}(t) + (D_{d_g} + G_g)\dot{q}(t) + K_g q(t) = f_u(t) + f_g(t) + Ls(j)n(q(t), t) \quad (1)$$

can be accepted as linear system. Here, the index g denotes the whole system. The equation (1) is able to be discretized into N(=7) sub - finite systems and its equation of motion with crack at some subsystem j is described by

$$i_e = 1, \ldots, N \quad (2)$$

$$j_k(i_e) = [(i_e - 1)\frac{n}{2} + 1]_{(i_e:1,\ldots,N)} \quad (3)$$

$$i = j_k, \ldots, j_k + n - 1 \quad (4)$$

$$j = j_k, \ldots, j_k + n - 1 \quad (5)$$

With i_e, j_k, i and j the vector in explicit form and the equation of motion can be given as follows :

$$q_{(i_e+1)(i)_{(i=1,\ldots,\backslash frac\,n\,2+1)}} = q_{(i_e-1)(\backslash frac\,n\,2+i)} \quad (6)$$

$$\sum_{i_e=1}^{N} \sum_{j_k=1}^{j_k(i_e)+n-1} [M_e \ddot{q}_{j_k(i_e)}(t) + (D_e + G_e) \times \dot{q}_{j_k(i_e)}(t) + K_e q_{j_k(i_e)}(t)] = [f_u(t)](i_e)_{(i_e=3,4,5)} + f_g(t)] \times (i_e)_{(i_e=1,\ldots,N)} + Ls(n_f, i_e) \times [n(q_{(t)}(t), t)]_{(i_e=1,\ldots,N)} \quad (7)$$

where the index erepresents the elementary subsystem. The elementary notations in the equations denote as follows :

- $q(t),\ \dot{q}(t),\ \ddot{q}(t)$: displacement vector,velocity vector and acceleration of the system.
- M_g, K_g : mass matrix, stiffness matrix of undamaged section.
- $D_{d_g},\ G_g = -G^{T_g}$: matrix of the damping and gyroscopic matrix.
- $q_e(t),\ \dot{q}_e(t),\ \ddot{q}_e(t)$: displacement vector, velocity vector and acceleration of the elementary sub systems. $q_e(t) \in \mathrm{Re}^n$, n(=8), and nn(=32) are degree of freedom of considered elementary sub system and total system. The $q_e(t)$ consists of $q_e(t) = (x_l, y_l, \theta_{x_l}, \theta_{y_l}; x_r, y_r, \theta_{x_r}, \theta_{y_r})$, the indices l and r denote the left and right node and $(x_r, y_r, \theta_{x_r}, \theta_{y_r})$ are the coordinates at the subsystem.
- $f_u(t), f_g(t), n(q(t), t)$: vector of unbalance, gravitation input vector, and vector of the nonlinearities caused by unexpected influence(crack).
- M_e, K_e : mass matrix, stiffness matrix of undamaged secton.
- $D_{d_e}, G_e = -G^{T_e}, Ls_{(n_f, i_e)}$: matrix of the damping, gyroscopic effects and distribution vector with regard to the crack at subshaft number i_e

All system matrices are constant in terms of time t[9,10, 11] and the distribute matrix[11,12] is given in the following way:

$$Ls(i_e) = \begin{bmatrix} 000, \ldots, 1000 & , \ldots, & 000 \\ 000 & , \ldots, & 1000, \ldots, 000 \end{bmatrix}^T \quad (8)$$

From now on the index j will be left out with respect to the whole dynamic system.

It is normally convenient for further operation to write the equation above via state space notation with $x(t) = [q(t)^T, \dot{q}(t)^T]$ including the nonlinearities of the motion created by a crack and under assumption that it concerns random disturbance in plant with $s(t)$.

$$\dot{x}(t) = Ax(t) + Bu(t) + N_R n_R(x(t)) + Ws(t) \quad (9)$$

The equation of the measurement is given by

$$y = Cx(t) + w_m(t) \quad (10)$$

where A is $(N_n \times N_n)$ dimensional system matrix which is responsible for the system dynamic with $N_n = 2nn, u(t)$ denotes r-dimensional vector of the excitation inputs due to gravitation and unbalances and C presents $(m_e \times N_n)$-dimensional measurement matrix. W is the $(N_n \times N_n)$ dimensional matrix and $s(t)$ presents the plant vector of noise. w_m denotes the white measurement noise. x(t) is N_n- dimensional state vector, and y(t) is m_e- dimensional vector of measurements respectively.

Here, the vector $n_R(x(t))$ characterizes the n_f-dimensional vector of nonlinear functions due to the crack. N_R is the input matrix of the nonlinearties and the order of N_R is of $(Nn \times n_f)$. It is presupposed that the matrices A, B, C, N_R, the vector u(t) and y(t) are already known. On the assumption that the plant noise and measurement noise are uncorrelated[15,16], the expected value E over the arbitary pair fixed time index t, τ and the variance σ can be defined as follows:

$$E[w_s(t)w_s^T(\tau)] = \int_{-\infty}^{\infty} w_s(\tau)p(w_s)dw_s \cdot \int_{-\infty}^{\infty} w_s^T(\tau)p(w_s^T)dw_s^T \quad (11)$$

where $p(w_s)$ and $p(w_m)$ denote the Gaussian probability density functions. The expected value E in terms of the plant and measurement can be given by

$$E[w_m(t)w_m^T(\tau)] = \int_{-\infty}^{\infty} w_m(\tau)p(w_m)dw_m \cdot \int_{-\infty}^{\infty} w_m^T(\tau)p(w_m^T)dw_m^T \quad (12)$$

$$\sigma_s^2 = \int_{-\infty}^{\infty} w_s w_s^T(\tau)p(w_s)dw_s \quad (13)$$

$$\sigma_m^2 = \int_{-\infty}^{\infty} w_m w_m^T(\tau)p(w_m)dw_m \quad (14)$$

$$E[w_s(t)] = 0,\ E[w_m(t)] = 0, \quad (15)$$

$$E[w_s(t) \; w_m^T(t)] = 0, \tag{16}$$

$$E[w_s(t) \; w_s^T(\tau)] = Q_s(t)\delta(t-\tau), \tag{17}$$

$$Q_s(t) = Q_s^T(t), \tag{18}$$

$$E[w_m(t) w_m^T(\tau)] = R_m(t)\delta(t-\tau), \tag{19}$$

$$R_m(t) = R_m^T(t), \tag{20}$$

$$E[w_s(t) x_0^T] = 0, E[w_m(t) x_0^T] = 0, \tag{21}$$

where the weighting matrix Q corresponding to the plant and R_m regarding to the measurement should be suitably chosen by the trial and errors. Now it remains to reconstruct the unknown nonlinear vector $n_R(x(t), t)$ which mentions the disturbance force caused by a crack. The basic idea is to get the signals from $ n_{R}(x(t)) $ approximated by the linear fictitious model[7]

$$n_R(x(t), t) \approx H \; v(t) \tag{22}$$

$$\dot{v}(t) = V \; v(t) \tag{23}$$

$$\dim \; v(t) = s \tag{24}$$

that describes the time behaviour of the nonlinearities due to the appearance of the crack approximately as follows:

$$n_R(x(t), t) \approx \breve{n}_R(\breve{x}(t)) = H \breve{v}(t) \tag{25}$$

where $\breve{v}(t)$ follows from(29, see below).

The matrices H and V have to be chosen according to the technical background considered in terms of oscillator model or integrator model[6,7]. To make the signals $\breve{n}(\breve{x}(t))$ available, it needs the elementary observer(EOB) to be designed.

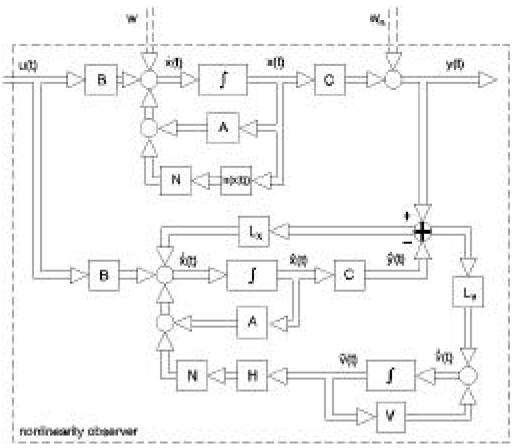

Fig. 2. Elementary observer : EOB.

At first the given system(\ref{tif}) has to be extended with the fictitious model(22, 23) into extended model

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} A & N_R H \\ 0 & V \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{u}(t) \tag{26}$$

$$y(t) = [\, C \;\vdots\; 0] \begin{bmatrix} x(t) \\ \cdots \\ v(t) \end{bmatrix} \tag{27}$$

Here, $N_R H$ couples the fictitious model(22, 23) to the whole system. To enable the successful estimates, it is obligatory to pay attention to the condition $m_e \geq n_f$. i.e, the number of the meas-urements must be at least equal or greater than the modelled nonlinearities. In the case the above requirements are satisfied, then the elementary observer in terms of an identity observer can be designed as follows:

$$\begin{bmatrix} \dot{\breve{x}}(t) \\ \dot{\breve{v}}(t) \end{bmatrix} = \begin{bmatrix} A - L_x C & N_R H \\ - L_x C & V \end{bmatrix} \begin{bmatrix} \breve{x}(t) \\ \breve{v}(t) \end{bmatrix}$$

$$\begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{u}(t) + \begin{bmatrix} L_x \\ L_v \end{bmatrix} y(t) \tag{28}$$

$$\breve{y}(t) = \begin{bmatrix} \breve{x}(t) \\ \cdots \\ \breve{v}(t) \end{bmatrix}, \tag{29}$$

where matrices L_x and L_v are the gain matrix of the observer and white noise vector related to the state measurement respectively. The above equation(28, 29) means that the observer consists of a simulated model with a correction feedback of the estimation error between real and simulated measurements. The matrix A_o has $(N_n + n_f \times N_n + n_f)$-dimension and represents the dynamic behaviour of the elementary observer. The asymptotic stability of the elementary observer can be guaranteed by a suitable design of the gain matrices L_x and L_v which are possible under the conditions of detectability or observabilty of the extended system(26, 27). The successfull estimation under the asymptotic stability the eigenvalue of the considerd observer (A_o) must be settled on the left side of the eigenvalue of the given system (A_e) to make the dynamic of the observer faster than the dynamic of the system.

The fictitious model of the crack behaviours is able to be designed using integrator model[11,12] based on the choosen crack model[1] as follows:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{30}$$

$$V = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{31}$$

$$n_{(R;1, x(t)_1)} \approx v_1(t) \tag{32}$$

$$n_{(R;2, x(t)_2)} \approx v_2(t) \tag{33}$$

The observer gain matrices L_x and L_v can be calculated by pole assignment or by the Riccati equation[6,7] as follows:

$$0 = A + P + P \; A^T - P \; C^T R_m^{-1} C \; P +$$

$$\begin{bmatrix} W \\ \cdots \\ 0 \end{bmatrix} Q \; [W^T \;\; 0] \tag{34}$$

$$\begin{bmatrix} L_x \\ \cdots \\ L_u \end{bmatrix} = P \; C^T \; R_m^{-1} \tag{35}$$

The weighting matrix Q and R_{m} have to be suitably chosen by the trial and errors.

III. Design of an Estimator for the localization

In the above section it has been studied how to design the elementary observer (EOB) for the detection at a given local position. It means that a certain place on the shaft is initially given as the position of a crack. In the real running operation there is not any information about the position of the crack, so the elementary observer has to survey not only the assigned local position but also any other place on the shaft and give the signals whether a crack exists or not. As it has been known, it is possible to detect the crack assigned certain place on the shaft. In case a crack appears at any subsystem in running time, it must be detected as well. But in many cases it has been shown that it is impossible or very difficult to estimate the position of the crack at all subsystem on the shaft with one EOB. Generally it depends on the number of the subsystem, the number of EOB. For the estimation of a crack position a method based on Estimator is designed. The main idea is to feel the related crack forces from a certain local position to the arranged elementary observer. This is the main task in this section.

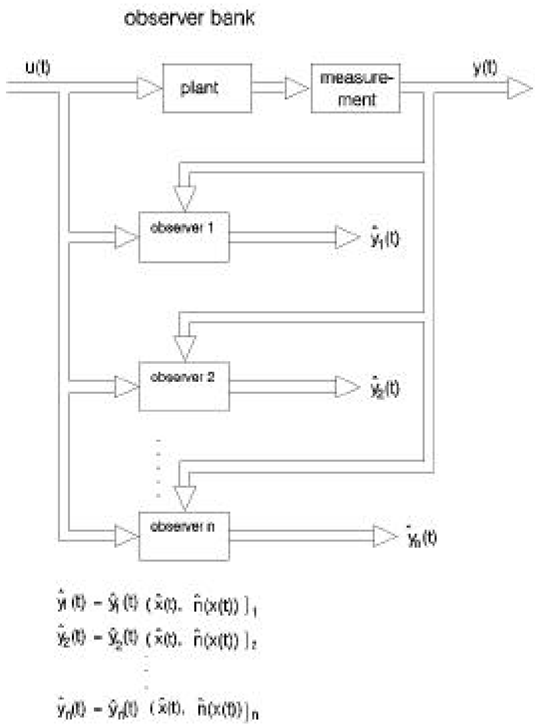

Fig. 3. Estimator(Observer bank).

Fig. from 2-1 shows the structure of the Estimator (Observer Bank) considered.

It consists of a few elementary observers. The number of elementary observer depends on the number of the subsystems modelled. Every elementary observer which is distinguished from the distribution matrix $Ls_{(i_e)}$ gets the same input(excitation) u(t) and the feedback of the measurements and is going to be set up at a suitable place on the given system. For the appreciate arrangement of the EOB, the distribution matrix on the analogy of (8) has been applied. In this way the Estimator(observer bank) is established with the EOB. To estimate the local place of the crack, there are two steps. First of all, the EOB must be observable to certain local in the meaning of the asymptotical stability in the system. The requirement has been satisfied by the criteria from Hautus[6,7]

$$Rank\begin{vmatrix} \lambda I_{N_n} - A & -N_R(L_{s_{(i)}})\,H \\ 0 & \lambda I_{nf} - V \\ C_e & 0 \end{vmatrix}$$

$$= \dim(x_e(t)) + \dim(v(t))$$

$$= N_n + n_f (= s) \qquad (36)$$

This means that the EOB has to be capable of estimating the crack at any location, where EOB is situated on the given system.

The unknown crack position is to be found by the EOB arranged in a certain local place with the related crack forces resulting from the crack. To guarantee this the condition(36) is supposed to be fulfilled. In this work two EOB are arranged at the 2nd subsystem and the 6th like this:

$$Ls_{(2)}(i=2) = 1,\ \text{otherwise}\ \ Ls_{(2)}(i) = 0$$
$$Ls_{(6)}(i=30) = 1,\ \text{otherwise}\ \ Ls_{(6)}(i) = 0.$$

The equation of the estimator with the first EOB A at the 2nd subsystem

$$\begin{vmatrix} \dot{\hat{x}}(t)_{(2)} \\ \dot{\hat{v}}(t)_{(2)} \end{vmatrix} \begin{bmatrix} A - L_x C & N_R(L_{(s)}) \\ -L_v C & V \end{bmatrix}$$
$$\begin{bmatrix} \hat{x}(t)_{(2)} \\ \hat{v}(t)_{(2)} \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{u}(t) +$$
$$\begin{bmatrix} L_x \\ L_v \end{bmatrix} (y(t) + w_m) \qquad (37)$$

and the 2nd EOB B at the 6th is described by

$$\begin{vmatrix} \dot{\hat{x}}(t)_{(6)} \\ \dot{\hat{v}}(t)_{(6)} \end{vmatrix} \begin{bmatrix} A - L_x C & N_R(L_{(6)}) \\ -L_v C & V \end{bmatrix}$$
$$\begin{bmatrix} \hat{x}(t)_{(6)} \\ \hat{v}(t)_{(6)} \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{u}(t) +$$
$$\begin{bmatrix} L_x \\ L_v \end{bmatrix} (y(t) + w_m) \qquad (38)$$

IV. Example

The Estimator consists of two EOB. The first EOB A is situated at the 2nd subsystem and the 2nd EOB B is placed at the 6th subsystem. As an example the given crack is at the 3rd and 4th of the node in the system considered.

The figures from 3-1 presents the results of the theoretical investigation without the effects of noise. and show the crack forces at 3rd and 4th of the node in vertical direction respectively. By the comparison of the force scale in the figure from 3-1, the EOB A see the force from the crack given at the 3rd and 4th of the node. In this way the Estimator estimates the existence of a crack by thr crack force and localize its position according to the height. The figure from 3-2 shows corrupted signals due to random

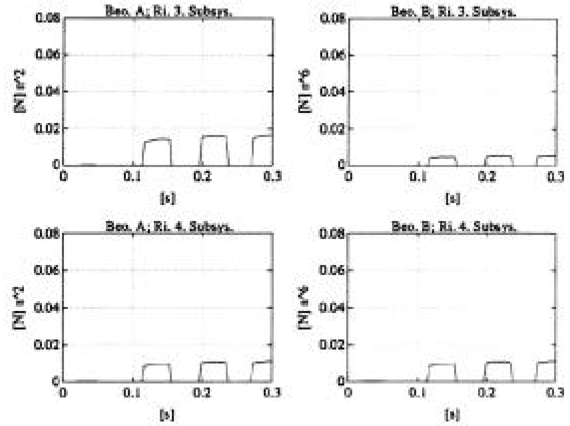

Fig. 4. EOB A, B : crack in 3rd and 4th Subsystem, $t_{(ri;1)} = 0.135$, $t_{(ri;2)} = 0.15$, $t_{(s)} = 0.03[s]$, Y coordinate : crack force in N , X coordinate : time in [sec], i = 1,7; j= 1, 2.

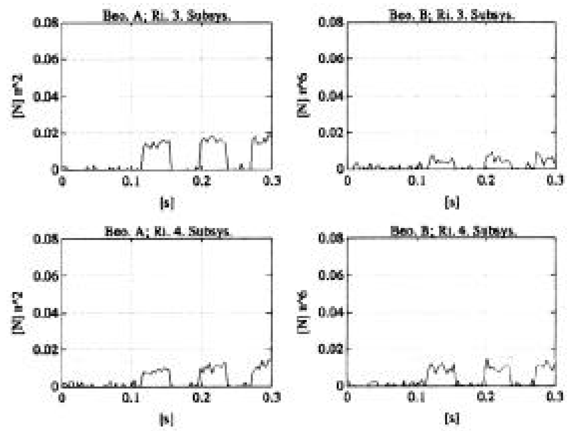

Fig. 5. EOB, A, B : crack in the 3rd and 4th Sub-system, $t_{(ri)} = 0.15$, $t_{(s)} = 0.03[s]$.

disturbance in plant. It also means to estimate the local crack position under constant depth with respect to crack forces. These forces related from certain position of crack to EOB A and EOB B are supposed to be interpreted as mechanical forces due to the breathing and gaping from the Gash model[1]. The numerical value of the ρ_q concerned with the weighting matrix Q are of

$Q_{(i,j;i=j=1,\ldots,32)} = 10$,

$Q_{(i,j;i=j=33,\ldots,62)} = 15$,

$Q_{(i,j;i=j=63)} = 2*10^4$,

$Q_{(i,j;i=j=64)} = 10^{4.25}$ respectively.

The factor ρ_r of the weighting matrix R_m is of 0.975 and $diagR_{(i,j)}$ is of 1. The matrices Q and R_m have been chosen by the trial and errors.

It has been noticed that the observer estimates the signals very well. The external signal exists in case of the opened crack. On analogy of the system model, the minimal and maximal values depend on the depth if only the crack is situated at the position where the EOB are located. Otherwise the position of the crack plays a part in the values of the forces regarding to the excited inputs as well.

However, the crack forces are a clear indicator for the appearance of a crack in operating time. The other figures which have been left out because of quantity of this paper, show that EOB B which is arranged at the right bearing, is not able to estimate the crack in 1st subsystem. In the simulation the given depth is of 2 mm and the time of appearance of the crack makes 0.2 sec.

V. Summary and conclusions

Using FEM the mathematical model of the rotating shaft including a crack has been presented. Based on the mathematical model including plant random disturbance, the elementary observer and an estimator have been developed. With this Estimator the task of the crack detection and localization have been done. The above methods give a clear relation between the damaged shaft by a crack and the caused phenomena in vibration by means of the measurement at both bearings. Successful theoretical results have been given. The forces in the results are the internal forces, which have been reconstructed as disturbance forces created by the crack.

From the given Example, it has been theoretically shown, that the cracks on the shaft can be detected. The Estimator is able to estimate the location of a crack. It has been also shown that it is possible to detect a crack and to localize the crack position under the random plant noise and the clean measurement. The method considered can be applicated in the similar area of nonlinear dynamic effect from a crack in terms of the suitable design of an Estimator. Anyway, the suggested methods are very significant not only for the further theoretical research and development but also for the transfer in experiments.

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Appendix

Using the abbreviation $ii = i - j_k + 1, jj = j - j_k + 1$ the sum of the matrices with accordance to equations (2) and (3) can be described as follows :

$$M^{(g)}_{(jk,jk)}(ie) = \sum_{ie=1}^{N} \left[\sum_{jk=1}^{(ie-1)\frac{n}{2}+1} \left(\sum_{i,j=jk}^{jk+n-1} M_e(ii,jj) \right) \right]_{(ie)} + M^0_{(\dim e \times \dim e)} \tag{39}$$

$$K^{(g)}_{(jk,jk)}(ie) = \sum_{ie=1}^{N} \left[\sum_{jk=1}^{(ie-1)\frac{n}{2}+1} \left(\sum_{i,j=jk}^{jk+n-1} K_e(ii,jj) \right) \right]_{(ie)} + K^0_{(\dim e \times \dim e)} \tag{40}$$

$$G^{(g)}_{(jk,jk)}(ie) = \sum_{ie=1}^{N} \left[\sum_{jk=1}^{(ie-1)\frac{n}{2}+1} \left(\sum_{i,j=jk}^{jk+n-1} G_e(ii,jj) \right) \right]_{(ie)} + G^0_{(\dim e \times \dim e)} \tag{41}$$

$$D^{(g)}_{(jk,jk)}(ie) = \sum_{ie=1}^{N} \left[\sum_{jk=1}^{(ie-1)\frac{n}{2}+1} \left(\sum_{i,j=jk}^{jk+n-1} D_e(ii,jj) \right) \right]_{(ie)} + D^0_{(\dim e \times \dim e)} \tag{42}$$

The matrices used in equation(\ref{tif}) are follows

$$A = \begin{bmatrix} 0 & \vdots & I_{(nn)} \\ \cdots & \cdots & \cdots \\ -(M_g)^{-1} K_e & \vdots & -(M_g)^{-1}(D_{dg} + G_g) \end{bmatrix} \tag{43}$$

The index i denotes the number of the subsystem. The vector of the order of the excitation and the matrix of nonlinearites,

$$\tilde{u}(t) = \begin{bmatrix} 0 \\ \cdots \\ M_g^{-1} f_e \end{bmatrix}_{(64 \times 1)} \tag{44}$$

$$N_R(L_{s(i)}) = \begin{bmatrix} 0 \\ \cdots \\ -M_g^{-1} L_{s(i)} \end{bmatrix}_{(64 \times 1)} \tag{45}$$

is of (64 x 1). where the vector of the excitation consists of graviation and harmonic unbalance , is presented by

$$f_e = f_{(g, i_e; i=1,\ldots,N)} + f_{(u, i_e = 3,4,5)} \tag{46}$$

$$f_{(g;2)} = f_{(g;30)} = 0, \tag{47}$$

$$f_{(g;6)} = f_{(g;10)} = f_{(g;14)} = f_{(g;18)} = f_{(g;22)} = f_{(g;26)} = -mg, \tag{48}$$

The order of the f_g is of (32 x 1) and f_u is of (32 x 1).

$$f_{(u;17)} = f_{(u;21)} = f_{(u;25)} = -e_m \Omega^2 m_{(ex)} \sin(\Omega t + \beta) \tag{49}$$

$$f_{(u;18)} = f_{(u;22)} = f_{(u;26)} = e_m \Omega^2 m_{(ex)} \cos(\Omega t + \beta) \tag{50}$$

where angle of the phase: $\beta = 0$, length of the subsystem of rotor $el = 2m$, Diameter of the subsystem of rotor makes $ed = 0.25m$. The mass of elemental subsystem: $m = \pi el \rho \frac{ED^2}{4}$, The density is of $\rho = 7860 \frac{kg}{m^3}$ excentricity: $e_m = 0.0001$, mass of the excentricity: $m_{(ex)} = 3$ m respectively. The modulus $E i_f$ is of $2.1 * 10^5 N/mm^2$. The stiffness of bearing: $K_{beaing} = 15 * 10^5 N/mm^2$. The easurement matrix of order(4 x 64), $C_{(i=1,\ldots 4, j=1,\ldots,64)} = 0$,

except $C_{(1 \times 1)} = C_{(2 imes 2)} = C_{(29 \times 29)} = C_{(30 \times 30)} = 1$.

The number of the nonlinearities n_f are of 1 and the number of the measurements m_e makes 4. The elementar matrices K_e, M_e and D_g which depend on the geometry, are given in[4,5,6].

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