Redundancy Resolution by Minimization of Joint Disturbance Torque for Independent Joint Controlled Kinematically Redundant Manipulators

Myoung Hwan Choi

Abstract : Majority of industrial robots are controlled by a simple independent joint control of joint actuators rather than complex controllers based on the nonlinear dynamic model of the robot manipulator. In this independent joint control scheme, the performance of actuator control is influenced significantly by the joint disturbance torques including gravity, Coriolis and centrifugal torques, which result in the trajectory tracking error in the joint control system. The control performance of a redundant manipulator under independent joint control can be improved by minimizing this joint disturbance torque in resolving the kinematic redundancy. A 3 DOF planar robot is studied as an example, and the dynamic programming method is used to find the globally optimal joint trajectory that minimize the joint disturbance torque over the entire motion. The resulting solution is compared with the solution obtained by the conventional joint torque minimization, and it is shown that joint disturbance can be reduced using the kinematic redundancy.

Keywords : disturbance torque, independent joint control, redundant manipulator

I. Introduction

Majority of the present day industrial robots are controlled by independent joint control schemes. The independent joint control schemes assume that a robot consists of independent linear joint control subsystems with fixed load inertia, rather than a multi-variable control system as in model based control approaches ([1]-[5]). Independent joint control scheme is described in Fig.1b, and the model based control scheme is described in Fig.1a. In the independent joint control schemes, the effects of varying effective inertia, Coriolis and centrifugal torques, and gravity are regarded as disturbance torque that is introduced into the control system. This control scheme is easy, inexpensive, robust and reliable. For this reason, this control scheme is widely used in industrial robot systems. The performance of this type of controller depends on the ability of the joint controller to reject the joint disturbance torque. This method gives a good performance when the joint speeds of the robot are low and the dynamic coupling between the joints can be ignored. However, as the motion of the joints become faster, the performance of this control scheme deteriorates because the effects of the joint disturbance torque also increase, and results in the joint trajectory tracking error. Thus, for a high speed - high accuracy motion, it is important that the joint disturbance torque be minimized at the motion planning stage. A formulation and application of the joint disturbance torque for the independent joint controlled manipulator was reported in [6], where the joint disturbance torque was defined and used in the trajectory planning of a non-redundant manipulator such that the joint disturbance torque generated during the straight line motion was minimized.

A robotic manipulator is said to be kinematically redundant when it has more joints than necessary for executing a task. For a n degrees of freedom manipulator in a m dimensional task space, if $n>m$, it is kinematically redundant. Since the joint space has greater degrees of freedom than the task space, for a given end effector location, there exist infinite number of joint solutions. In order to select a joint solution from the infinite number of candidate solutions, thus resolving the kinematic redundancy, an optimization criterion is chosen and the joint solution that minimizes the optimization criterion is selected. Hence, by an appropriate choice of the optimization criteria, the redundant manipulator can be used to perform a secondary task. For example, the joint velocity minimization can be used as the optimization criterion to minimize the unwanted joint motion [7], joint limit avoidance can be used to maintain the joint position near the center of the position range [8], the distance from obstacles can be used to execute a motion among obstacles without collision [9], singularity measures can be used to avoid singularities [10], kinetic energy can be used to minimize the power consumption [11], and joint torque minimization can be used to reduce the torque magnitude so that actuators with small torque bounds can be used to perform a heavy task [12].

The performance quality of the redundant manipulator, on the other hand, depends on the quality of the joint motion control. The optimization criteria mentioned above is concerned with the motion planning stage. No considerations are given to the joint control schemes, and the question of control performance is not addressed. When a manipulator is controlled by the independent joint control scheme, the performance is affected by the joint disturbance torques, as noted above, and it is important that the joint disturbance torque be minimized at the motion planning stage. Hence, the kinematic redundancy can be exploited to reduce this joint disturbance torque. In this paper, the minimization of joint disturbance torque is proposed as a new optimization criterion for the kinematic redundancy

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resolution when the independent joint control is employed. By minimizing the proposed criterion, the joint disturbance torque introduced to the independent joint control system is minimized at the motion planning stage, and as a consequence, the joint tracking error can be reduced during the actual motion. This is a motion planning scheme suitable for high speed - high accuracy motion of the manipulator.

In this work, the explicit expressions for the joint disturbance torque of a 3 DOF planar redundant manipulator is obtained using the method presented in [6]. Then, dynamic programming method is employed to find the globally optimal joint trajectory to produce a straight line motion that minimize the joint disturbance torque over the entire motion. The resulting solution is compared with the solution obtained by minimizing the conventional joint torque, and it is shown that the joint motion obtained by the proposed method results in smaller joint disturbance torques.

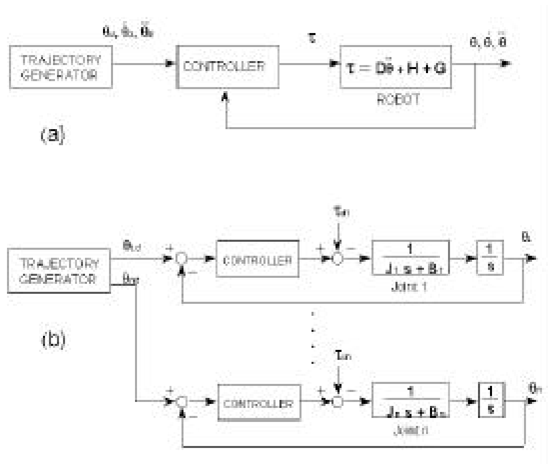

Fig. 1. Block Diagram for (a) model based control and (b) independent joint control.

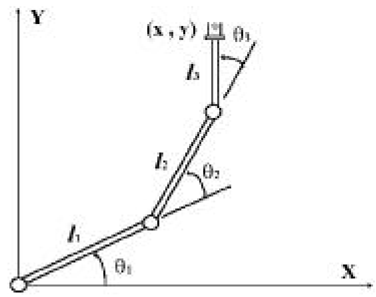

Fig. 2. Three link planar manipulator.

II. Formulation of joint disturbance torque

The basis of the independent joint control is that a robot is viewed as a set of independent actuator-load pairs, where the load inertia of joints are unknown constants. The joint disturbance torque, τ_{di} , are unknown and varies depending on the motion of the joints. For n joint robotic manipulator, the dynamic equation of motion can be expressed by the following Lagrange-Euler equation

$$\tau = D(\theta)(\ddot{\theta}) + H(\theta, \dot{\theta}). \tag{1}$$

where $D = (D_{ij})$ is an n x n inertia matrix, $H = (h_i)$ is a n x 1 vector containing Coriolis, centrifugal effect and gravity loading vector, and τ is a n x 1 joint torque vector. If the robot manipulator is viewed as n independent joint control systems, the dynamic equation in (1) can be written as

$$\hat{D}_{ii}\ \ddot{\theta}_i + \tau_{di} = \tau_i \ ,\ i = 1, \cdots, n \tag{2}$$

where

$\hat{D}_{ii}$ = sum of components of D_{ii} which are not dependant on θ (3)

$$\tau_{di} = \sum_{j=1, j \neq i}^{n} D_{ij}(\theta)\ddot{\theta}_j + (D_{ii} - \hat{D}_{ii})\ddot{\theta}_i + h_i(\theta, \dot{\theta}). \tag{4}$$

By definition, $\hat{D}_{ii}$ are not related with the joint position, velocity, and acceleration, and are constant regardless of the motion of the joints. $\hat{D}_{ii}$ are the constant inertial load for the joint i, and corresponds to J_i in Fig. 1b. In (4), τ_{di} are the disturbance torque of joint i and contains torque components in robot dynamic equation related to the variation of effective joint inertia due to joint position change, Coriolis and centrifugal effect, and gravity loading.

The joint disturbance torque in (4) is applied to a 3 DOF planar robot. In Fig. 2, a simplified model of a 3 DOF planar robot is shown. It is assumed that planar arm moves in a horizontal plane, and the gravity acceleration vector is set to zero. It is also assumed that the links have uniform mass distribution. In this figure, m_i, l_i, θ_i are mass, length and joint position of the i-th link respectively. The Lagrange Euler equation of motion for this manipulator can be written in a below form [13]

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \tag{5}$$

and from the definitions in (3) and (4), $\hat{D}_{11}$, $\hat{D}_{22}$, $\hat{D}_{33}$, τ_{d1}, τ_{d2}, τ_{d3}, can be written as below.

$$\hat{D}_{11} = (l_1^2 m_1)/3 + l_1^2 m_2 + (l_2^2 m_2)/3 + l_1^2 m_3 + l_2^2 m_3 + (l_3^2 m_3)/3 \tag{6}$$

$$\hat{D}_{22} = (l_2^2 m_2)/3 + l_2^2 m_3 + (l_3^2 m_3)/3 \tag{7}$$

$$\hat{D}_{33} = (l_3^2 m_3)/3 \tag{8}$$

$$\begin{aligned} \tau_{d1} = {} & [m_2 l_1 l_2 C_2 + 2m_3 l_1 l_2 C_2 + m_3 l_2 l_3 C_3 \\ & + m_3 l_1 l_3 C_{23}]\ddot{\theta}_1 + [(1/3)m_2 l_2^2 + m_3 l_2^2 \\ & + (1/3)m_3 l_3^2 + (1/2)m_2 l_1 l_2 C_2 + m_3 l_1 l_2 C_2 \\ & + m_3 l_2 l_3 C_3 + (1/2)m_3 l_1 l_3 C_{23}]\ddot{\theta}_2 \\ & + [(1/3)m_3 l_3^2 + (1/2)m_3 l_2 l_3 C_3 \\ & + (1/2)m_3 l_1 l_3 C_{23}]\ddot{\theta}_3 \\ & - m_3 l_3 \dot{\theta}_1 \dot{\theta}_3 (l_2 S_3 + l_1 S_{23}) - m_3 l_3 \dot{\theta}_2 \dot{\theta}_3 \\ & (l_2 S_3 + l_1 S_{23}) - (1/2)(m_3 l_3 \dot{\theta}_3^2 (l_2 S_3 + l_1 S_{23})) \\ & - \dot{\theta}_1 \dot{\theta}_2 l_1 (m_2 l_2 S_2 + 2m_3 l_2 S_2 + m_3 l_3 S_{23}) \end{aligned}$$

$$-(1/2)(\dot{\theta}_2^2 l_1(m_2 l_2 S_2 + 2m_3 l_2 S_2 + m_3 l_3 S_{23})) \quad (9)$$

$$\begin{aligned}\tau_{d2} = {} & [(1/3)m_2 l_2^2 + m_3 l_2^2 + (1/3)m_3 l_3^2 \\ & + (1/2)m_2 l_1 l_2 C_2 + m_3 l_1 l_2 C_2 + m_3 l_2 l_3 C_3 \\ & + (1/2)m_3 l_1 l_3 C_{23}]\ddot{\theta}_1 + [m_3 l_2 l_3 C_3]\ddot{\theta}_2 \\ & + [(1/3)m_3 l_3^2 + (1/2)m_3 l_2 l_3 C_3]\ddot{\theta}_3 \\ & - (1/2)(m_3 l_2 l_3 \dot{\theta}_1\dot{\theta}_3 S_3) - (1/2)(m_3 l_2 l_3 \dot{\theta}_2\dot{\theta}_3 S_3) \\ & - (1/2)(m_3 l_2 l_3 \dot{\theta}_3^2 S_3) + (1/2)(l_1 \dot{\theta}_1^2 \\ & (m_2 l_2 S_2 + 2m_3 l_2 S_2 + m_3 l_3 S_{23}))\end{aligned} \quad (10)$$

$$\begin{aligned}\tau_{d3} = {} & [(1/3)m_3 l_3^2 + (1/2)m_3 l_2 l_3 C_3 \\ & + (1/2)m_3 l_1 l_3 C_{23}]\ddot{\theta}_1 + [(1/3)m_3 l_3^2 \\ & + (1/2)m_3 l_2 l_3 C_3]\ddot{\theta}_2 + (1/2)(m_3 l_3 \\ & (l_2 \dot{\theta}_1^2 S_3 + 2l_2\dot{\theta}_1\dot{\theta}_2 S_3 + l_2\dot{\theta}_2^2 S_3 + l_1\dot{\theta}_1^2 S_{23}))\end{aligned} \quad (11)$$

Let τ_d denote the joint disturbance vector such that

$$\tau_d = [\tau_{d1}\ \tau_{d2}\ \tau_{d3}]^T \quad (12)$$

$\breve{D}_{11}$, $\breve{D}_{22}$, $\breve{D}_{33}$ in (6), (7) and (8) are components of D_{11}, D_{22}, D_{33} respectively, which are independent of the joint motion, and these can be used as the constant joint inertia in the independent joint controlled manipulators. Similarly, τ_{di} in (9), (10), (11) are the components of the joint torques that depend upon the joint motion, and thus correspond to the joint disturbance torques.

III. Redundancy resolution by global minimization of joint disturbance torque

In this section, the joint disturbance torque formulated in section II is used in the kinematic redundancy resolution so that the optimal joint trajectory for a given end effector motion is obtained which globally minimizes the joint disturbance torque. The state equation formulation for the dynamic programming presented in [14] is applied in this work, and it is briefly described in this section. The global optimization problems can be represented in the following general form in optimal control theory [15]

$$\begin{aligned} & \min \int P(z, u, t)dt \quad (13) \\ & \text{s.t.}\ \dot{z} = s(z, u, t) \\ & z^- \le z \le z^+ \\ & u^- \le u \le u^+ \end{aligned}$$

where P is the optimization criterion, z is the system state, and u is the control input.

The kinematic relations between the joint position θ and the task variable x of a manipulator can be expressed as follows.

$$x = f(\theta) \quad (14)$$

$$\dot{x} = J(\theta)\dot{\theta} \quad (15)$$

$$\ddot{x} = J(\theta)\ddot{\theta} + \dot{J}(\theta, \dot{\theta})\dot{\theta} \quad (16)$$

where x is the m-dimensional vector describing the end effector in task space, $J \in R^{m\times n}$ represents the Jacobian matrix. In order to formulate the state equation, the dynamic equation of the manipulator is parameterized by (n-m) independent joint variables. Let the joint position and torque vectors be partitioned as

$$\theta = \begin{bmatrix}\hat{\theta} \\ \bar{\theta}\end{bmatrix}, \quad \tau = \begin{bmatrix}\hat{\tau} \\ \bar{\tau}\end{bmatrix} \quad (17)$$

where $\hat{\theta}$ and $\bar{\theta}$ are dependent and independent joint variables respectively, and $\hat{\tau}$ and $\bar{\tau}$ are the corresponding joint torques. The dependent joint variable $\hat{\theta}$ can be given as $\hat{\theta} = \hat{\theta}(\bar{\theta}, x_d, t)$, a function of independent joint variable $\bar{\theta}$, and desired end effector path x_d through the kinematic relation (14). Let the Jacobian matrix be partitioned as $J = [\hat{J}_{m\times m}\ \bar{J}_{m\times(n-m)}]$, then the dependent joint velocities $\dot{\hat{\theta}}$ is given by (15) as $\dot{\hat{\theta}} = \hat{J}^{-1}(\dot{x}_d - \bar{J}\dot{\bar{\theta}})$. The state is chosen as $z \equiv [\bar{\theta}^T\ \dot{\bar{\theta}}^T]^T$, and the control input is chosen as $\bar{\tau}$. Then, the state equation is given by

$$\frac{d}{dt}\begin{bmatrix}\bar{\theta} \\ \dot{\bar{\theta}}\end{bmatrix} = \begin{bmatrix}\dot{\bar{\theta}} \\ [O_{(n-m)\times m}\ I_{(n-m)}]\, D^{-1}(\tau - H)\end{bmatrix} \quad (18)$$

The joint torque τ is obtained as follows. The expression (16) can be rewritten using (1) as

$$A(\theta)\tau = b(\theta, \dot{\theta}, \ddot{x}_d) \quad (19)$$

where $A(\theta) = JD^{-1}$, $b(\theta, \dot{\theta}, \ddot{x}_d) = \ddot{x}_d - \dot{J}\dot{\theta} + JD^{-1}H$. Let A be partitioned as $A = [\hat{A}_{m\times m}\ \bar{A}_{m\times(n-m)}]$, then the dependent joint torques $\hat{\tau}$ can be obtained as

$$\hat{\tau} = \hat{A}^{-1}(b - \bar{A}\bar{\tau}) \quad (20)$$

thus the derivation of the state equation is completed. Note that the kinematic relationships were used to reduce the number of states and the number of control variables to 2(n-m) and (n-m) respectively.

In this work, n = 3 and m = 2, so there are two states and one control variable. The independent joint variable was selected as $\bar{\theta} = \theta_3$, hence the state variables used were $z_1 = \theta_3$, $z_2 = \dot{\theta}_3$. The control variable was selected as $\bar{\tau} = \tau_3$, The dynamic programming technique [15] can now be applied to this optimal control problem.

Table. 1. Parameters used in dynamic programming.

	lower limit	upper limit	step size
$z_1 = \theta_3$	-0.5π	$+0.5\pi$	0.00021 (rad)
$z_2 = \dot{\theta}_3$	$-\pi$	$+\pi$	0.063 (rad/s)
$u = \tau_3$	- 10	+ 10	0.2 (N/m)
t	0	1.01	0.003(sec)

IV. Simulations and discussions

For the demonstration of kinematic redundancy resolution

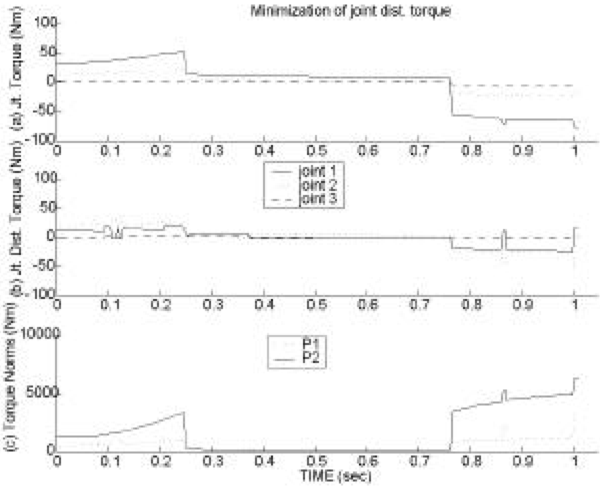

Fig. 3. Result of Joint Disturbance torque minimization. (a) Joint torque, (b) Joint disturbance torque, (c) P1 and P2.

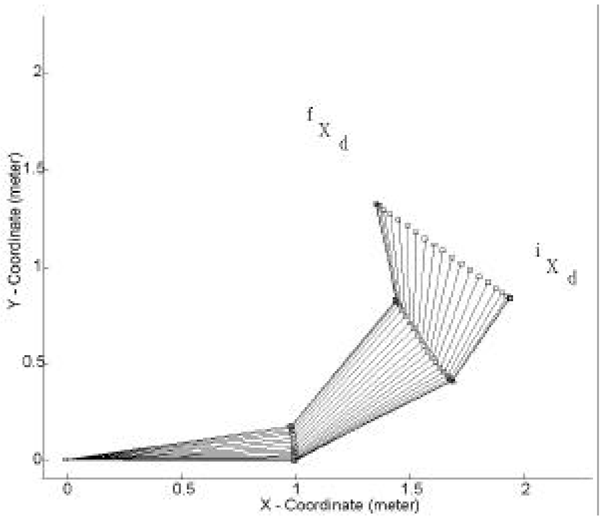

Fig. 4. Motion trajectory for joint disturbance torque minimization.

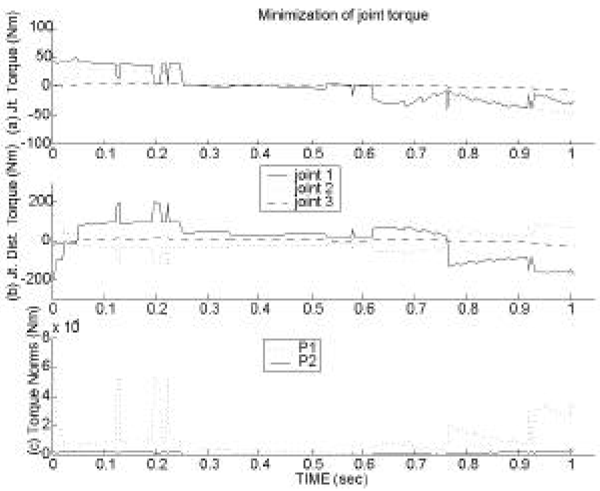

Fig. 5. Result of joint torque minimization. (a) Joint torque, (b) Joint disturbance torque, (c) P1 and P2.

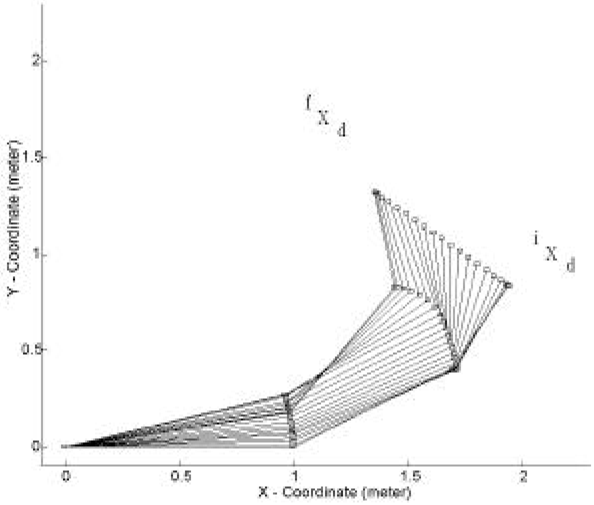

Fig. 6. Motion trajectory for joint torque minimization.

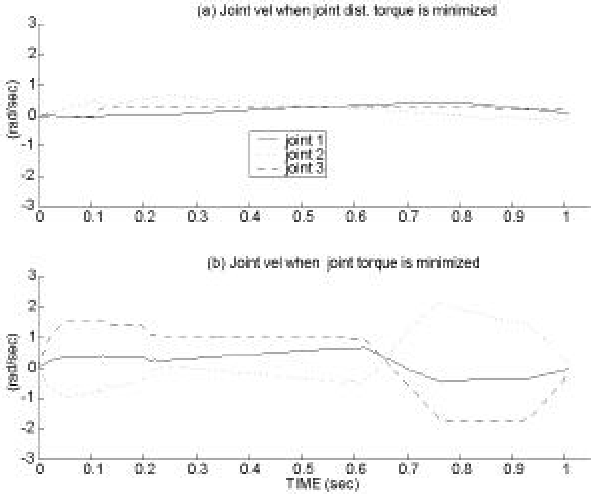

Fig. 7. (a) Joint trajectory when P1 is minimized, (b) Joint trajectory when P2 is minimized.

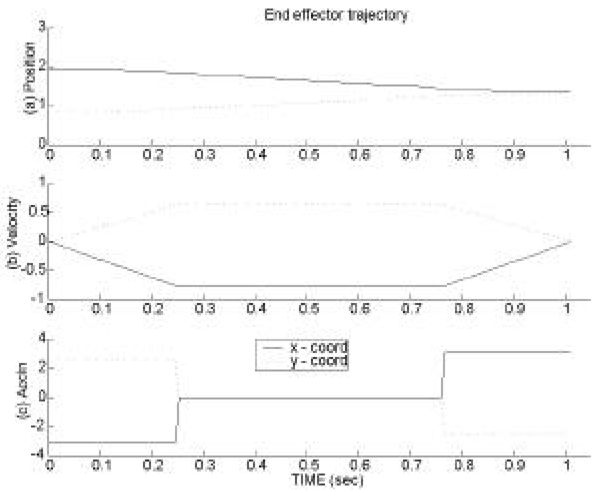

Fig. 8. End effector trajectory along a straight line. (a) Position(meter), (b) Velocity (meter/sec), (c) Acceleration (meter/sec2).

using global joint disturbance torque minimization, the three link planar robot in Fig. 2 is used in this work. The link parameters used are $l_1 = 1.0m$, $l_2 = 0.8m$, $l_3 = 0.5m$, $m_1 = 10$**kg**, $m_2 = 8$**kg**, $m_3 = 5$**kg**. The manipulator task is to move from a initial end effector position of ${}^{i}x_d = (1.94, 0.83)^T$ with the joint position of $\theta_i = [0\ 30\ 30]^T$ degrees to the final end effector position of ${}^{f}x_d = (1.36,\ 1.32)^T$, in a straight line trajectory. The position and velocity profile of the end effector along the straight line path is shown in Fig. 8. The joint velocities are zero at initial and final positions, and the end effector travels through a distance of 0.76 meters in 1.01 sec.

The optimization criterion used is the 2-norm square of the joint disturbance torque vector, P_1, formulated as follows. For comparison, the 2-norm square of joint torque vector, P_2 is also used.

$$P_1 = \|\tau_d\|^2, \quad P_2 = \|\tau\|^2 \tag{21}$$

For dynamic programming, the joint space was discretized as shown in Table 1. In order to reduce the memory requirement to the minimum, for each node, only the optimal control input from a node in a stage to the optimal node in the next stage was recorded. The cost of the node was not stored for the nodes in all stages. Instead, the cost of nodes of two stages, the present and next stages, were stored and used to compute the optimal control from present stage to the next stage. The total number of nodes were 4.443x108, each control input required 2 bytes memory storage, and the total memory requirement for the node data storage was 899.85 Mbytes.

The optimal solution minimizing the joint disturbance torque, i.e., P_1 is shown in Fig. 3. The joint torques are shown in (a), and the joint disturbance torques are shown in (b). The resulting values of $P_1 = \|\tau_d\|^2$ and $P_2 = \|\tau\|^2$ are shown in (c). The resulting joint trajectory is shown in Fig.4. For comparison, the optimal solution minimizing the conventional joint torque, i.e., $P_2 = \|\tau\|^2$ are shown in Fig.5, and corresponding joint trajectory is shown in Fig.6. The simulation result shows that the peak value of joint disturbance torque, $P_1 = \|\tau_d\|^2$ shown in Fig.3c is an order of magnitude less than that shown in Fig. 5c. The joint velocity of the two trajectories are shown in Fig. 7. This figure shows that minimizing the joint disturbance torque, i.e., $P_1 = \|\tau_d\|^2$ results in a smaller joint motion.

In Fig. 3 and Fig. 5, discontinuities can be seen in the joint torques. The discontinuities in the control input of an optimal control problem can be seen elsewhere such as in bang-bang control input of a time optimal problem. This phenomenon can be reduced by including the joint torque continuity condition in the problem formulation (13), and/or reducing the step sizes in Table 1. Reducing the step sizes, however, could lead to the explosion of the amount of the required memory, so in practice, it can not be reduced arbitrarily small, and a compromise has to be made.

It can be seen that the kinematic redundancy can be used to reduce the joint disturbance torque while executing the specified end effector motion. The reduction of the joint disturbance torque is important when the joints are controlled by the independent joint control scheme, since the joint disturbance torque causes joint tracking error.

V. Conclusions

In an independent joint control scheme of a manipulator, the robot system is regarded as a set of independent actuator-load pairs, and the joint interaction such as centrifugal and Coriolis torque and the gravity effect are regarded as joint disturbance torque introduced into the joint control system. This joint disturbance torques results in the joint trajectory tracking error, and degrades the control performance. It is necessary to minimize the joint disturbance torque at the motion planning stage , and the kinematic redundancy of a manipulator can be exploited to reduce the joint disturbance torque. In this paper, the minimization of the joint disturbance torque is proposed as a new optimization criterion for the kinematic redundancy resolution when the independent joint control scheme is employed. Using a 3 DOF planar manipulator as an example, a globally optimal joint trajectory to produce a straight line motion of the end effector is obtained that minimizes the joint disturbance torque during the motion. The result is compared with a solution obtained by minimizing the conventional joint torque, and it is shown that using the proposed criterion, a joint motion with reduced joint disturbance torque is possible.

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