Longitudinal Spacing Control of Vehicles in a Platoon[1]

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Abstract: The Lyapunov stability theorem is used to derive a control law that can be used to control the spacing between vehicles in a platoon. A third order system is adopted to model the vehicle and power-train dynamics. In addition, the concept of "Expected Spacing Error" is introduced and used to form a Lyapunov function. Then a control law that always decreases the Lyapunov function is selected. A platoon of four vehicles and various scenarios are used to demonstrate the performance of the proposed control law. Simulation results show that the slinky effects are essentially removed and the robustness to the uncertainty of vehicle dynamics is substantially improved over previous results.

Keywords: Lyapunov function, vehicle control, longitudinal control, platoon

I. Introduction

It has been shown that the concept of operating vehicles in a platoon can significantly improve the efficiency of existing road systems. In order to meet such expectations, it is necessary to develop advanced vehicle control systems (AVCS) which form a part of intelligent vehicle/highway systems (IVHS)[1]-[4]. As shown in Fig. 1, one of the many functions of AVCS is longitudinal control, in which the pre-assigned spacing H_i between the predecessor, M^{i-1} and the follower vehicle, M^i in the platoon is to be tightly maintained. Most of the earlier works assume that the follower vehicle M^i has access to the velocity v_L and acceleration a_L of the platoon lead vehicle M^L and can therefore, measure the relative spacing, relative velocity and relative acceleration between itself and the vehicle in front of it[5]-[8].

There are many strategies for designing longitudinal control laws. To cite a few, PID type controllers have been proposed by Shladover [5][6], Sheikholeslam [7][8], and Fujioka et al [9]. Hedrick et al. [10][11] have extensively used the sliding mode control methodology.

In the literature cited above, the general structure of the control input a_i^c to vehicle M^i may be written as

$$a_i^c = f(x_{i-1}, v_{i-1}, a_{i-1}, x_i, v_i, a_i, v_L, a_L) \tag{1}$$

where(x_i, v_i, a_i) denote the absolute position, velocity, and acceleration of vehicle M^i. It has been reported that 1 m or less spacing can be maintained in a platoon[5]-[9] by using such controllers. Sheikholeslam and Desoer [12].

showed that tight spacing control is still possible with no communication of information about the lead vehicle.

In this paper, we propose another form of longitudinal control law, viz.,

$$a_i^c = f(x_{i-1}, v_{i-1}, a_{i-1}, x_i, v_i, a_i, a_{i-1}^c) \tag{2}$$

which completely eliminates the need for a communication link between the platoon's leading vehicle and the follower vehicles, and uses the local link between the predecessor and follower vehicles, as depicted in Fig. 2. We will show that the procedure used in this paper for designing control laws somewhat mitigates the ad-hoc nature of selecting the variables for feedback or the structure of the sliding surface. Also, it effectively accommodates wide variations in vehicle dynamics and is suitable for the spacing control of mixed vehicles.

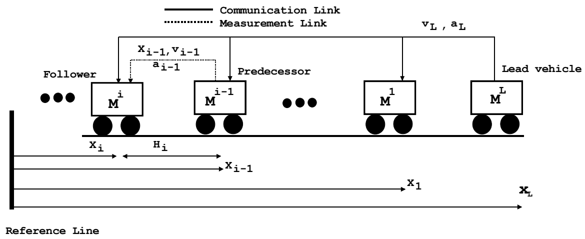

Fig. 1. Platoon with global link.

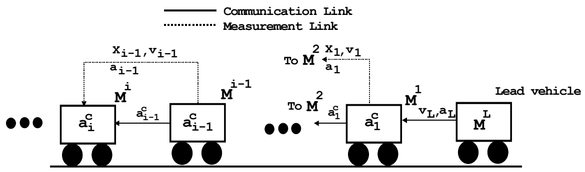

Fig. 2. Platoon with local link.

II. Vehicle dynamics model

Various models for vehicle dynamics have been used in the study of longitudinal control of platoons. For a platoon traveling with a constant speed and direction, we have adopted the following third-order model [5][6],

$$\dot{x}_i = v_i \tag{3}$$

$$\dot{v}_i = a_i \tag{4}$$

$$\dot{a}_i = \frac{1}{\tau_i}\left(a_i^c - a_i \right) . \tag{5}$$

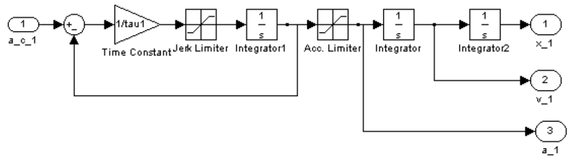

Fig. 3. Block diagram of vehicle dynamics.

where τ_i, the time constant of the vehicle propulsion system, is a single parameter which represents the characteristics of different vehicles. Shown in Fig. 3 is the block diagram of vehicle dynamics. Jerk and acceleration limits are easily

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considered with this model.

Referring to Fig. 1, the error in spacing distance between vehicles M^{i-1} and M^{i} can be written as

$$\delta_i = x_{i-1} - x_i - H_i . \tag{6}$$

Then $\dot{\delta}_i$ and $\ddot{\delta}_i$ represent the relative velocity and acceleration, respectively.

III. Longitudinal controller design

The objective of longitudinal control is to maintain the spacing error δ_i below a predetermined level or, if possible, at zero. For example, the PID type controller may be written in its general form as

$$a_i^c = k_x\, \delta_i + k_v\, \dot{\delta}_i + k_a\, \ddot{\delta}_i + k_v^L\,(v_L - v_i) + k_a^L\,(a_L - a_i) \tag{7}$$

where k_x, etc, are controller gains. With this control law, the control command is determined using the information measured and/or estimated at the current time. In other words, the controller is always working, as long as there is a spacing error at the current time. The only time the controller stops working is when the following condition is satisfied:

$$0 = k_x\, \delta_i + k_v\, \dot{\delta}_i + k_a\, \ddot{\delta}_i + k_v^L\,(v_L - v_i) + k_a^L\,(a_L - a_i) . \tag{8}$$

However, if some predictive nature is incorporated into the controller, one can make it behave more intelligently. Even if current δ_i is large, if it is expected to be decreasing, no further control action would be necessary. By contrast, some action is required when δ_i is expected to increase even though its current magnitude is below the predetermined level.

With the above reasoning in mind, we can introduce the concept of "expected spacing error" as follows:

$$d_i = (x_{i-1} - x_i) + (v_{i-1} - v_i)\, t_{go} + \frac{1}{2}\,(a_{i-1} - a_i)\, t_{go}^2 \tag{9}$$

For the sake of brevity, we assumed H_i is zero. As one may easily understand, d_i is the spacing error at time t_{go} if both the predecessor and follower vehicles keep their respective current accelerations constant for t_{go}. Therefore, d_i reflects not only the current spacing error but also the relative velocity and acceleration. One may let

$$t_{go} = t_f - t \tag{10}$$

where t is the current time and t_f is the unspecified future time.

In this paper, we have used the Lyapunov stability theorem [13] and the expected spacing error to obtain the control laws. First, let's define a positive definite scalar function, that is, the Lyapunov function, as shown below.

$$V_i = \frac{1}{2}\, d_i^2 . \tag{11}$$

Differentiating the above Lyapunov function gives

$$\dot{V}_i = d_i \dot{d}_i \tag{12}$$

and using Eq. (9)

$$\begin{aligned} \dot{d}_i &= (\dot{x}_{i-1} - \dot{x}_i) + (\dot{v}_{i-1} - \dot{v}_i)\, t_{go} - (v_{i-1} - v_i) + \frac{1}{2}\,(\dot{a}_{i-1} - \dot{a}_i)\, t_{go}^2 - (a_{i-1} - a_i)\, t_{go} \\ &= \frac{1}{2}\,(\dot{a}_{i-1} - \dot{a}_i)\, t_{go}^2 \end{aligned} \tag{13}$$

since we have assumed engine and powertrain dynamics as in Eq. (5), Eq. (13) becomes

$$\dot{d}_i = \frac{1}{2}\left[\frac{1}{\tau_{i-1}}\,(a_{i-1}^c - a_{i-1}) - \frac{1}{\tau_i}\,(a_i^c - a_i) \right] t_{go}^2 . \tag{14}$$

To use Lyapunov stability theorem, we select a_i^c so that

$$\dot{d}_i = -N_i\, d_i \tag{15}$$

where N_i is a positive constant.

Then, Eq. (12) may be rewritten as

$$\dot{V}_i = -N_i\, d_i^2 \tag{16}$$

which is a negative definite function. This implies that the Lyapunov function V_i is a strictly decreasing function, meaning that Lyapunov stability is assured. Finally, the controller which satisfies the above argument is

$$a_i^c = \frac{\tau_i}{\tau_{i-1}}\,(a_{i-1}^c - a_{i-1}) + a_i + \frac{2\, N_i\, \tau_i}{t_{go}^2}\, d_i \tag{17}$$

where t_{go} and N_i are left undetermined as design parameters.

While the proposed control law still needs to measure the spacing error and relative speed/acceleration, the main difference comparing with previous control laws is that the follower vehicles do not receive any information directly from the platoon lead vehicle except the first one. Instead, the unidirectional communication link is required, for each follower vehicle to receive the control command from the preceding vehicle. Another feature of the proposed control law is that it systematically accommodates the wide variation in dynamic characteristics of the vehicles participating in the platoon. This feature will be beneficial if a platoon is composed of vehicles of various kinds. Therefore, the follower vehicle has full access to the predecessor vehicle for information such as control commands and dynamic characteristics. Further, due to the cascaded structure of the control laws, the follower vehicle knows what the current situation is, and what all the vehicles in front of it are about to do.

IV. Simulation and performance analysis

As shown in Fig. 4, a four-vehicle platoon is chosen and used in the simulation because it must retain the essential characteristics of a platoon. Individual vehicles are assumed to be identical and the design parameters t_{go} and N_i are selected after some numerical experiments. Just for comparison purposes, a PID type controller given by Eq. (7) and a simulation scenario were adopted from reference [6]. The relevant numbers used in the simulation are summarized in Table 1. The spacing distance H_i is set at zero for ease of presenting the simulation results. Then, the control objective is that every vehicle lines up with the lead vehicle. This should make no difference in interpreting the simulation results.

Table 1. Vehicle and controller characteristics.

Vehicle Characteristics :	$\tau_i = 0.1$ sec
PID Controller :	$k_x = 3.6, k_v = 0.9, k_a = 0.0, k_v^L = 2.4, k_a^L = 0.0$
Lyapunov Controller :	$t_{go} = 1.0, \quad N_i = 10$

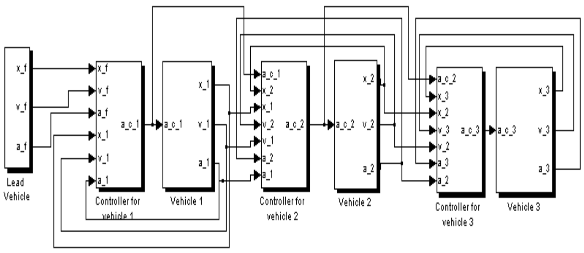


Fig. 4. Platoon simulation environment.

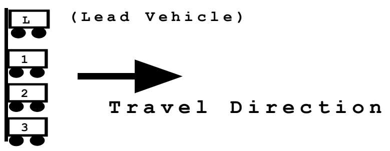
1. Response to abrupt speed changes by the lead vehicle

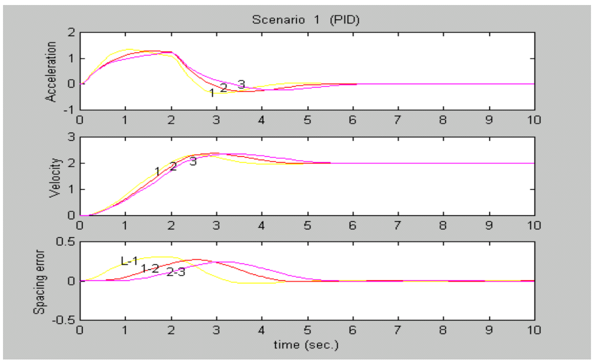
In this example, the lead vehicle accelerates at $1m/\sec^2$ to speed up by $2m/\sec$ after 2 seconds and there are no initial spacing errors. As one may see from Fig. 5, the PID-type controller performs just as human drivers do. In the early stage, the spacing error between the lead and first vehicle is the largest. That is because of the delay in propulsion or engine dynamics. Although the first vehicle notices the acceleration of the lead vehicle earlier than the other vehicles behind it, the difference in acceleration and speed are still great, producing a large spacing error. But the spacing error between the next two vehicles gets smaller because the difference in acceleration and speed have not yet built up enough to yield noticeable errors. However as time goes by, the first vehicle catches up with the lead vehicle faster than any of the other vehicle and spacing errors disappear after about 6 seconds.

However, the Lyapunov controller proposed in this paper shows exceptional performance in the sense that all vehicles move together and achieve almost identical acceleration and speed with each other. In this case, the follower vehicle knows what its predecessors will do, and takes the same control action. Therefore, the spacing errors remain virtually unchanged.

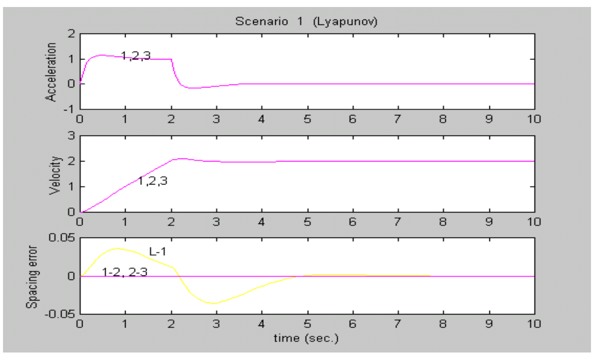
2. Response to initial spacing errors

In this scenario, all the vehicles are moving with the same speed as that of the lead vehicle, and there are initial spacing errors, as shown in Fig. 6. To line up with the lead vehicle, it would be best if the first vehicle decelerated, the second kept the current speed (no control action), and the third vehicle accelerated. Unfortunately, the structure of the PID controller does not allow that sort of intelligent maneuver. Because the first vehicle is ahead of the platoon leader, it will decelerate and because the second vehicle is behind the first, it will ac-



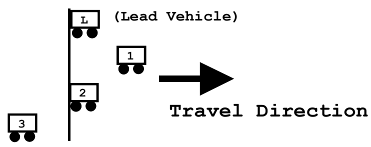


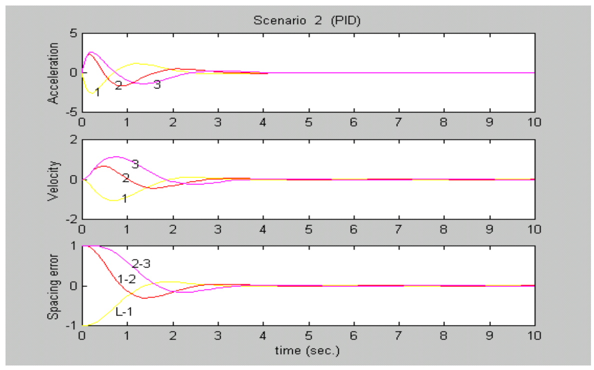
(a) PID controller



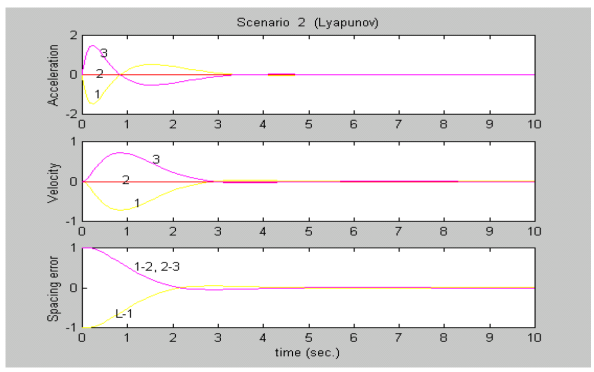
(b) Lyapunov controller

Fig. 5. Response to sudden speed change.





(a) PID controller



(b) Lyapunov controller

Fig. 6. Response to initial spacing errors.

celerate. This will make the third vehicle accelerate at a higher speed than the second vehicle. The simulation results clearly support this expectation.

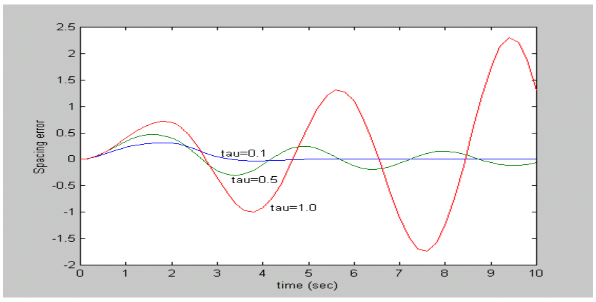
However, the Lyapunov controller completely eliminates the need for second vehicle speed changes and the first and the third vehicles adjust their speed in a symmetric pattern using much lower acceleration/deceleration levels than with the PID controller. By avoiding any unnecessary acceleration/deceleration, the Lyapunov controller has the effect of saving energy. It also should be noted that oscillations in response are virtually removed.

3. Sensitivity to vehicle dynamics variations

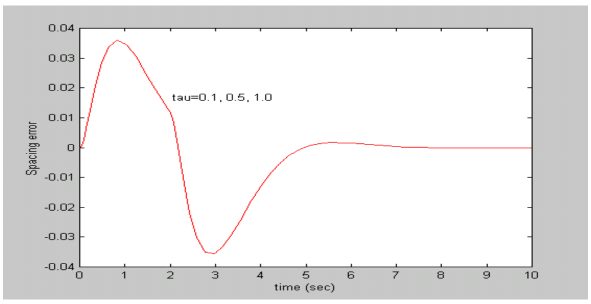
Most longitudinal controllers are designed using some nominal vehicle dynamics -either linear or nonlinear. McMahon and Hedrick[14] reported that first order lag model approximations to vehicle dynamics including engine and powertrain involve a significant amount of simplification and many assumptions. The time constant of first-order models may be a function of many factors such as vehicle speed and mass. Therefore, the controller must be designed so that it can accommodate, or at least be robust to, variations in vehicle dynamics. Shladover[6] has suggested the use of a gain scheduling technique. Alternatively, velocity-dependent characteristics in the first-order model may be taken into account during the design process[15].

The Lyapunov controller proposed in this paper inherently adapts itself to variations in the dynamics of the predecessor and follower vehicles. As one can see from Eq. (17), the time constants for both vehicles explicitly appear in the acceleration command.

The adaptive feature of the current controller may be shown by a simulation example. Using the same scenario used in the sudden speed change simulation and fixing all relevant gainis



(a) PID controller



(b) Lyapunov controller

Fig. 7. Effects of changes in powertrain time constant.

and parameters, only the time constant is varied. In Fig. 7, the spacing error between the lead vehicle and the first follower vehicle is presented. The performance of the PID controller degrades as the vehicle dynamics get slower, but the Lyapunov controller shows no performance degradation. For a platoon composed of different types of vehicles, the longitudinal controller of individual vehicles may need to be redesigned, but the Lyapunov approach provides a straightforward and unified procedure for obtaining a controller.

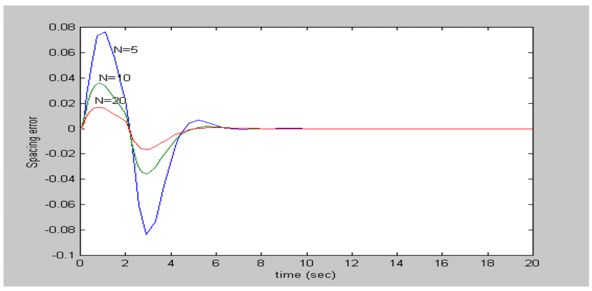
4. Effects of design parameters: N_i and t_{go}

In the present controller, N_i and t_{go} are left undetermined and regarded as design parameters which are to be selected. Although their theoretical effects on the performance of the proposed controller are not treated in this paper, some rough idea of them may be obtained from Eq. (17). Rewriting Eq. (17) in its expanded form using Eqs. (6) and (9), we get

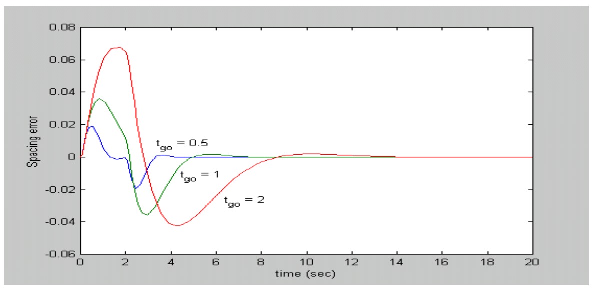
$$a_i^c = \frac{\tau_i}{\tau_{i-1}}(a_{i-1}^c - a_{i-1}) + a_i + 2\,N_i\,\tau_i\,(\frac{\delta_i}{t_{go}^2} + \frac{\dot{\delta}_i}{t_{go}} + \frac{1}{2}\ddot{\delta}_i)\,. \quad (18)$$

It may be argued that N_i plays the role of overall controller gain and t_{go} acts like a weighting factor between the current spacing error δ_i and its rate of change $\dot{\delta}_i$. In other words, a small t_{go} implies a more tight control of spacing errors. Therefore, it would be natural to put more weight on the current error. Also a large t_{go} means looser control of spacing errors, and a consequent longer time needed to reduce the errors.

Fig. 8 shows the simulation results as N_i and t_{go} vary. As before, the same scenario was used and only the spacing error between the lead and follower vehicles are displayed. As one can easily see, the overshoot is reduced as N_i increases. Although any choice of N_i and t_{go} assures overall stability, too small N_i may lead to the "apparent" instability of spacing errors between the lead vehicle and the first vehicle. That



(a) t_{go} = 1 sec.



(b) N_i = 10

Fig. 8. Effects of design parameters, N_i and t_{go}.

is because the jerk characteristics or the equivalent acceleration profile of the platoon leader vehicle is not considered in the design of controller for the first vehicle, and too small N_i means almost no control authority. It can seen that t_{go} affects both the overshoot and the speed of response.

5. Mixed Controllers

In case of communication link failure, the spacing of the whole platoon will be jeopardized and therefore, the system's reliability is very important. While the PID controller requires a global link in the sense that every follower vehicle should receive information from the lead car, the Lyapunov controller is wholly dependent on the local links. As suggested in Fig. 9, using both control approaches will mean that they will serve as a backups to each other because it is easy to switch from on controller to another regardless of which controller is used as the primary one. One should note that the mixing of two different kinds of control logic is possible because although the Lyapunov controller in the follower vehicle needs to receive the acceleration command a_{i-1}^c from the preceding car, it is not affected by the kind of procedure that is involved in obtaining it.

To illustrate this, two cases were simulated using the sudden speed change scenario. First, the PID controller is used as the primary controller while, the second car uses the Lyapunov controller. In the second case, the Lyapunov controller is used as the primary controller while the second car uses the PID controller. As can be noted from Fig. 10, the overall performance seems not to be degraded and the Lyapunov controller outperforms the PID in terms of spacing errors.

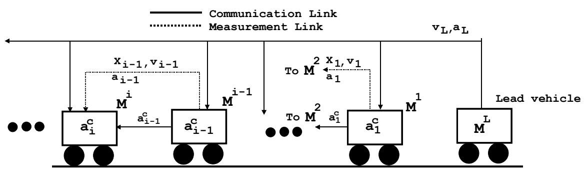
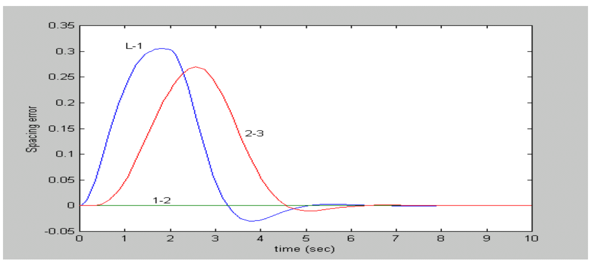
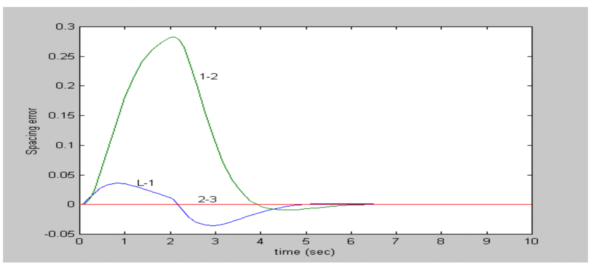


Fig. 9. Platoon with global and local communication link.



(a) Primary: PID, Backup: Lyapunov



(b) Primary: Lyapunov, Backup: PID

Fig. 10. Response of mixed controllers.

V. Conclusions and remarks

In this paper, the concept of "Expected Spacing Error" was introduced and the Lyapunov stability theorem was applied to the design of control logic that is used to control inter-vehicular spacing in a platoon. The proposed controller uses a local communication link between the predecessor and follower vehicles. Due to the cascaded structure of the controller, a vehicle in the platoon has full access to information on the dynamics and control of all the vehicles ahead of it. Numerical simulation was used to compare the performance of the Lyapunov controller with that of a PID-type controller. The simulation results showed that slinky effects could be completely removed and that the Lyapunov controller is robust to variations in vehicle dynamics. Also, the effect of design parameters on the performance of the Lyapunov controller was investigated.

For further research, the effects of jerk and acceleration limits need to be studied because they are not considered in the control design and are known to induce system instability. Also, the scope of the present work does not include the effects of sensor noise and actuator delays. Finally, the possibility of applying the results of this research to nonlinear vehicle dynamics needs to be addressed and the method of selecting control parameters warrants further in-depth investigation.

References

[1] Special Issue on Intelligent Vehicle Highway Systems, *IEEE Trans. on Vehicular Technology*, vol. 40, no. 1, 1991.

[2] P. Varaiya, "Smart cars on smart roads: problems of control," *IEEE Transactions on Automatic Control*, vol. 38, no. 2, pp. 195～07, Feb., 1993.

[3] J. K. Hedrick, M. Tomizuka, and P. Varaiya, "Control issues in automated highway systems," *IEEE Control Systems*, pp. 21～32. Dec., 1994.

[4] S. E. Shlodover, "Review of the state of development of advanced vehicle control systems," *International Journal of Vehicle System Dynamics*, vol. 24, pp. 551～595, 1995.

[5] S. E. Shladover, "Longitudinal control of automated guideway transit vehicles within platoons," *Journal of Dynamics Systems, Measurement, and Control*, vol. 100, pp. 302～310, 1978.

[6] S. E. Shladover, "Longitudinal control of automotive vehicles in close-formation platoons," *Journal of Dynamics Systems, Measurement, and Control*, vol. 113, pp. 231～241, 1991.

[7] S. Sheikholeslam and C. A. Desoer, "Longitudinal control of a platoon of vehicles I: Linear model," *PATH Research Report UCB-ITS-PRR-89-3*, Aug., 19, 1989.

[8] S. Sheikholeslam, and C. A. Desoer, "Longitudinal control of a platon of vehicles III: Nonlinear model," *PATH Research Report UCB-ITS-PRR-90-1*, April, 1990.

[9] T. et al, Fujioka, "Longitudinal vehicle following control

for autonomous driving," *AVEC'96 International Symposium on Advanced Vehicle Control* at the Aachen University of Technology, pp. 1293～1304. 24th-28th of June, 1996.

[10] J. K. Hedrick, et al, "Longitudinal vehicle controller design for IVHS systems," *Proceedings of American Control Conference*, pp. 3107～3111, 1991.

[11] D. H. McMahon, et al, "Longitudinal vehicle controller design for IVHS: Theory and experiment," *Proceedings of American Control Conference*, pp. 1753～1757, 1992.

[12] S. Sheikholesam, and C. A. Desoer, "Longitudinal control of a platoon of vehicles with no communication of lead vehicle information," Proceedings of American Control Conference, pp. 3102～3106, 1991.

[13] J-J. E. Slotine, and W. Li, *Applied Nonlinear Control*, Prentice-Hall International Edition, 1991.

[14] D. H. McMahon, and J. K. Hedrick, "Longitudinal model development for automated roadway vehicles," *PATH Research Report UCB-ITS-PRR-89-5*, Oct., 1989.

[15] A. S. Hauksdottir and R. E. Fenton, "On the design of a vehicle longitudinal controller," *IEEE Transactions on Vehicular Technology*, vol. VT-34, no. 4, pp. 182～187, Nov., 1985.

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