Receding Horizon FIR Filter and Its Square-Root Algorithm for Discrete Time-Varying Systems

Pyung Soo Kim and Wook Hyun Kwon

Abstract: A receding horizon FIR filter is suggested for discrete time-varying systems, combining the Kalman filter with the receding horizon strategy. The suggested filter is shown to be an FIR structure that has many good inherent properties. The suggested filter is represented in an iterative form and also in a standard FIR form. The suggested filter turns out to be a remarkable deadbeat observer that is often robust against system and measurement noises. It is also shown that the suggested filter is an unbiased estimator irrespective of any horizon initial condition. For the amenability to parallel and systolic implementation as well as the numerical stability, a square-root algorithm for the suggested filter is presented. To evaluate performance, the suggested filter is applied to a problem of unknown input estimation and compared with the existing Kalman filter based approach.

Keywords: kalman filter, receding horizon strategy, FIR filter, deadbeat property, square-root algorithm

I. Introduction

Recently, Kwon *et al*. have suggested optimal finite impulse response (FIR) filters for state estimation in discrete-time systems [1][2]. It is known that the optimal FIR filter provides several advantages over existing optimal filters such as the Kalman filter [3], which has an infinite impulse response (IIR) structure. Due to the FIR structure, this filter is known to be often robust against temporary modeling uncertainties and against numerical errors that may degrade the performance in the case of the IIR structure filter [4][5].

However, general readers might find it hard to understand the derivation of complicated filter equations since the optimal FIR filter is obtained from a Fredhelm integral equation with some complex boundary indices. Moreover, since existing optimal FIR filters [1][2] have been presented basically for systems without external control inputs, it is necessary to extend the derivation to systems with control inputs if we intend to use them in feedback control problems.

To improve the above drawbacks, the receding horizon (RH) FIR filter for discrete time-invariant systems was developed recently by combining the well known Kalman filter with the receding horizon strategy [6]. Since it comes from a modification of the well known Kalman filter algorithm, it is easy to understand the filtering algorithms. In addition, since the RH FIR filter deals with stochastic systems with control inputs, it is possible to apply this filter to feedback control problems. Furthermore, in recent, the RH FIR filter has been shown to be a best linear unbiased estimation (BLUE) filter with FIR structures, which processes the finite measurements on the most recent horizon linearly, doesn't require *a priori* statistics information of the horizon initial state and has the properties of unbiasedness, minimum variance and efficiency [7]. However, since time-varying systems are quite often used for many practical applications such as detection, tracking and guidance in the aerospace industry, the RH FIR filter for time-varying systems is also very necessary.

Therefore, in the current paper, the RH FIR filter for discrete time-varying systems is suggested. The RH FIR filter will be first presented in an iterative form and then in a standard FIR form similar to existing optimal FIR filters. The standard FIR form of the RH FIR filter in the current paper provides simpler algorithms for obtaining filter gains than existing optimal FIR filters. It is shown that the RH FIR filter becomes a remarkable deadbeat observer when applied to noise-free systems. It is also shown that the RH FIR filter is an unbiased estimator irrespective of any horizon initial condition. The suggested RH FIR filter would be a BLUE for discrete time-varying systems.

In the recent decades, square-root algorithms for state estimation have been preferred for implementation of the Kalman filtering and smoothing formulas [8][10]. They have been found to have several advantages in terms of the numerical stability which improves computational reliability, and the amenability to parallel and systolic implementation which overcomes computational burden. Therefore, a square-root algorithm for the suggested RH FIR filter will be established in the current paper.

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Pyung Soo Kim, Wook Hyun Kwon: School of Electrical Engineering, Seoul National University

II. RH FIR filters

Consider a linear discrete time-varying state-space model with control inputs

$$x_{k+1} = A_k x_k + B_k u_k + G_k w_k, \tag{1}$$

$$y_k = C_k x_k + v_k \tag{2}$$

where $x_k \in R^n$ is a state, and $u_k \in R^l$ and $y_k \in R^q$ are a known input and a measured output, respectively. The initial state x_{k_0} is a random variable with a mean $\bar{x}_{k_0}$ and a covariance Σ_{k_0}. The system noise $w_k \in R^p$ and the measurement noise $v_k \in R^q$ are zero-mean white Gaussian and mutually uncorrelated. The covariances of w_k and v_k are denoted by Q_k and R_k, respectively. These noises are uncorrelated with the initial state x_{k_0}. Matrices are assumed to be bounded for simplicity.

It is well known that the following Kalman filtering algorithm provides a minimum variance state estimate $\hat{x}_k$, called the one-step predicted estimate of the system state x_k with control inputs [3]:

$$\hat{x}_{k+1} = A_k \hat{x}_k + [A_k P_k C_k^T (R_k + C_k P_k C_k^T)^{-1}](y_k - C_k \hat{x}_k) + B_k u_k, \tag{3}$$

$$P_{k+1} = A_k(P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} A_k^T + G_k Q_k G_k^T \qquad (4)$$

where $\hat{x}_{k_0} = \bar{x}_{k_0}$, and P_k is the error covariance of the estimate $\hat{x}_k$ and $P_{k_0} = \Sigma_{k_0}$.

When the covariance of the initial state is very large but not infinite, one usually uses an information filter form [8], assuming that A_k is nonsingular. We can define

$$\Omega_k \equiv P_k^{-1}, \quad \overline{\Omega}_k \equiv \Omega_k + C_k^T R_k^{-1} C_k$$

if P_k is nonsingular. Then, the equation (4) is written as

$$\Omega_{k+1} = [I + A_k^{-T}\overline{\Omega}_k A_k^{-1} G_k Q_k G_k^T]^{-1} A_k^{-T}\overline{\Omega}_k A_k^{-1} \qquad (5)$$

where $\Omega_{k_0} = \Sigma_{k_0}^{-1}$. Therefore, the information form of the Kalman filter (3) can be written as

$$\begin{aligned} \hat{x}_{k+1} &= A_k(P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} P_k^{-1}\hat{x}_k \\ &\quad + A_k(P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} C_k^T R_k^{-1} y_k + B_k u_k \qquad (6) \\ &= A_k\overline{\Omega}_k^{-1}\Omega_k\hat{x}_k + A_k\overline{\Omega}_k^{-1}\Omega_k C_k^T R_k^{-1} y_k + B_k u_k. \end{aligned}$$

The filter algorithm (6) uses all measurements from the initial time k_0 to provide the state estimate at the present time k.

We now introduce the receding horizon strategy to the above filter (6). The RH FIR filter at the present time k uses finite measurements on the horizon $[k-N, k]$ and discards past measurements outside the horizon. We shall write $k_N \equiv k - N$ for compactness. We will call the state at k_N the horizon initial state, denoted by x_{k_N}. As mentioned previously, the horizon initial state x_{k_N} is assumed to be unknown and thus the horizon initial condition $\hat{x}_{k_N}$ is anything at all. It follows from this that the horizon initial state must have an arbitrary mean and an infinite covariance, $\Sigma_{k_N} = \infty$. We rederive the filter (6) at the present time k from the horizon initial time k_N under the unknown horizon initial state. The filter at the time $k_N + i$ on the interval $0 \le i \le N-1$ will be denoted by $\hat{x}_{k_N+i|k}$. The filter (6) on the horizon $[k_N, k]$ then becomes

$$\begin{aligned} \hat{x}_{k_N+i+1|k} &= A_{k_N+i}\overline{\Omega}_{k_N+i|k}^{-1}\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} \\ &\quad + A_{k_N+i}\overline{\Omega}_{k_N+i|k}^{-1}\Omega_{k_N+i|k} C_{k_N+i}^T R_{k_N+i}^{-1} y_{k_N+i} \qquad (7) \\ &\quad + B_{k_N+i} u_{k_N+i}, \quad 0 \le i \le N-1 \end{aligned}$$

where the horizon initial condition $\hat{x}_{k_N|k}$ is anything at all and

$$\begin{aligned} \Omega_{k_N+i+1|k} &= [I + A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} G_{k_N+i} Q_{k_N+i} G_{k_N+i}^T]^{-1} \\ &\quad A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} \end{aligned} \qquad (8)$$

with the horizion initial condition $\Omega_{k_N|k} = 0$.

In discrete time-varying systems, it is known that the nonsingularity of $\Omega_{k_N+i|k}$ is guaranteed by uniformly complete observability of the system [11]. That is, $\Omega_{k_N+i|k}$ becomes a positive definite matrix for all $i \ge l_0$ if $\{A_k, C_k\}$ is uniformly completely observable. In the filter (7), we can thus note that $\Omega_{k_N+i|k}$ may be singular on the interval $0 \le i < l_0$ with $\Omega_{k_N|k} = 0$. In this case, $\overline{\Omega}_{k_N+i|k}$ can be singular, thus the filter (7) cannot be defined during this interval. We can avoid this problem by pre-multiplying both sides of (7) by (8) to obtain

$$\begin{aligned} &\Omega_{k_N+i+1|k}\hat{x}_{k_N+i+1|k} \\ &= [I + A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} G_{k_N+i} Q_{k_N+i} G_{k_N+i}^T]^{-1} \\ &\quad A_{k_N+i}^{-T}[\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} + C_{k_N+i}^T R_{k_N+i}^{-1} y_{k_N+i} \\ &\quad + \overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} B_{k_N+i} u_{k_N+i}] \end{aligned} \qquad (9)$$

with the horizon initial condition $\Omega_{k_N|k}\hat{x}_{k_N|k} = 0$. Since the inversion of matrix $\overline{\Omega}_{k_N+i|k}$ disappears in (9), the singularity problem does not occur. Then, in the following theorem, the RH FIR filter is derived from (9) and can always be defined irrespective of the singularity, whereas the filter (7) cannot.

Theorem 1: Assume that $\{A_k, C_k\}$ is uniformly completely observable. When the horizon initial state $x_{k_N|k}$ is assumed to be unknown, the RH FIR filter $\hat{x}_{k|k}$ for discrete time-varying systems is given for any $N \ge l_0$ as

$$\hat{x}_{k|k} = \Omega_{k|k}^{-1}\hat{\eta}_{k|k} \qquad (10)$$

where $\hat{\eta}_{k|k}$ is obtained from the following iterative forms:

$$\begin{aligned} \hat{\eta}_{k_N+i+1|k} &= [I + A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} G_{k_N+i} Q_{k_N+i} G_{k_N+i}^T]^{-1} A_{k_N+i}^{-T} \\ &\quad [\hat{\eta}_{k_N+i|k} + C_{k_N+i}^T R_{k_N+i}^{-1} y_{k_N+i} + \overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} B_{k_N+i} u_{k_N+i}], \qquad (11) \\ &\quad 0 \le i \le N-1 \end{aligned}$$

with the horizon initial condition $\hat{\eta}_{k_N+i|k} \mid \{i=0\} = \hat{\eta}_{k_N|k} = 0$ ■

In the above theorem, (11) is obtained from (9) using the subsidiary estimate defined as $\hat{\eta}_{k_N+i|k} \equiv \Omega_{k_N+i|k}\hat{x}_{k_N+i|k}$ and the horizon initial condition $\hat{\eta}_{k_N|k} = \Omega_{k_N|k}\hat{x}_{k_N|k}$. Fig. 1 shows the concept of suggested RH FIR filter to obtain the state estimate $\hat{x}_{k|k}$ in (10) at the present time k.

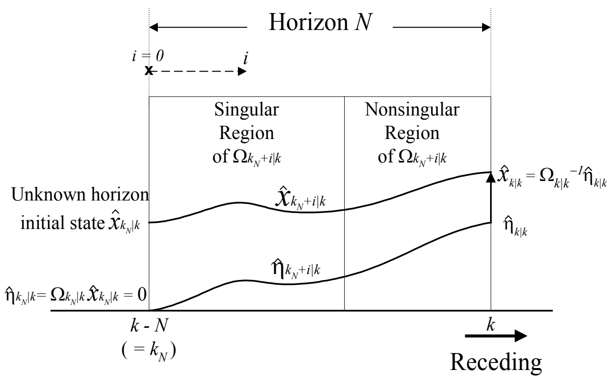

Fig. 1. Concept of RH FIR filter.

Although time-varying systems are often used in many areas such as detection, tracking and guidance in the aerospace industry, time-invariant systems are also used because of their simplicity. We therefore derive the RH FIR filter for time-invariant systems [6]. In time-invariant systems, the discrete Riccati equation (8) defined on the horizon $[k_N, k]$ are shift invariant and thus $\Omega_{k_N+i|k}$ is independent of k, denoted by Ω_i. In this case, the horizon initial condition $\Omega_{k_N|k}$ can be represented as Ω_0. Ω_i is obtained from (5) on the interval $0 \le i \le N-1$ as

$$\Omega_{i+1} = [I + A^{-T}\overline{\Omega}_i A^{-1} GQG^T]^{-1} A^{-T}\overline{\Omega}_i A^{-1},\ \Omega_0 = 0. \quad (12)$$

It is noted that $\Omega_N > 0$ for any $N \ge n$. Then, the RH FIR filter $\hat{x}_{k|k}$ for discrete time-invariant systems is given for any $N \ge n$ as

$$\hat{x}_{k|k} = \Omega_N^{-1}\hat{\eta}_{k|k} \quad (13)$$

where $\hat{\eta}_{k|k}$ is obtained from the following iterative forms:

$$\begin{aligned}\hat{\eta}_{k_N+i+1|k} &= [I + A^{-T}\overline{\Omega}_i A^{-1} GQG^T]^{-1} A^{-T} \\ &\quad [\hat{\eta}_{k_N+i|k} + C^T R^{-1} y_{k_N+i} + \overline{\Omega}_i A^{-1} B\, u_{k_N+i}]\end{aligned} \quad (14)$$

with the horizon initial condition $\hat{\eta}_{k_N|k} = 0$.

The suggested RH FIR filter provides several advantages. It is easy to understand since it comes from a modification of the well known Kalman filter algorithm. The suggested filter can always be defined irrespective of singularity problems caused by the infinite covariance of the horizon initial state. Since the suggested filter deals with stochastic systems with control inputs, it is possible to apply this filter to problems of feedback control. We can also expect that the suggested filter is often robust against temporary modeling uncertainties and against numerical errors that may cause a divergence phenomenon in the standard Kalman filter, since it utilizes only finite measurements on the most recent horizon.

III. Stadnard FIR form of RH FIR filter

The RH FIR filter (10) is an iterative form with the zero initial condition $\hat{\eta}_{k_N|k} = 0$. It is actually an FIR structure and thus can be represented in a standard FIR form similar to existing optimal FIR filters [1][2].

Define a transition matrix as

$$\begin{aligned}\Phi_{j+1,i} &= [I + A_j^{-T}\overline{\Omega}_{j|k} A_j^{-1} G_j Q_j G_j^T]^{-1} A_j^{-T}\Phi_{j,i}, \\ \Phi_{k_N+i,i} &= I, \quad k_N \le k_N + i \le j \le k-1\end{aligned} \quad (15)$$

where $\overline{\Omega}_{j|k}$ is obtained from (8). It is noted that $\Phi_{j,i}$ in (15) is a function of k since j varies on the interval $[k_N, k]$. At time k, (11) becomes

$$\hat{\eta}_{k|k} = \sum_{i=0}^{N-1}\Phi_{k,i} C_{k_N+i}^T R_{k_N+i}^{-1} y_{k_N+i} + \sum_{i=0}^{N-1}\Phi_{k,i}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} B_{k_N+i} u_{k_N+i}.$$

Since $\Omega_{k|k} > 0$, pre-multiplying both sides of by $\Omega_{k|k}^{-1}$ and defining filter gains $H_{k,i}, L_{k,i}$ as

$$H_{k,i} = \Omega_{k|k}^{-1}\Phi_{k,i} C_{k_N+i}^T R_{k_N+i}^{-1},\ L_{k,i} = \Omega_{k|k}^{-1}\Phi_{k,i}\overline{\Omega}_{k_N+i|k} A_{k_N+i}^{-1} B_{k_N+i}$$

yields a standard FIR form as

$$\hat{x}_{k|k} = \sum_{i=0}^{N-1} H_{k,i} y_{k_N+i} + \sum_{i=0}^{N-1} L_{k,i} u_{k_N+i}. \quad (16)$$

This is summarized in the following theorem.

Theorem 2: Assume that $\{A_k, C_k\}$ is uniformly completely observable. When the horizon initial state $x_{k_N|k}$ is assumed to be unknown, the RH FIR filter $\hat{x}_{k|k}$ can be represented in a standard FIR form (16) for any $N \ge l_0$. ■

The RH FIR filter (13) for time-invariant systems is now represented in a standard FIR form [6]. Due to the shift invariance of Ω_i in (12), a transition matrix can be defined on the finite interval $[0, N]$ instead of $[k_N, k]$ as

$$\begin{aligned}\Phi_{j+1,i} &= [I + A^{-T}\overline{\Omega}_j A^{-1} GQG^T]^{-1} A^{-T}\Phi_{j,i}, \\ \Phi_{i,i} &= I, \quad 0 \le i \le j \le N-1\end{aligned} \quad (17)$$

where $\overline{\Omega}_j$ is obtained from (12). At time k, (14) becomes

$$\hat{\eta}_{k|k} = \sum_{i=0}^{N-1}\Phi_{N,i} C^T R^{-1} y_{k_N+i} + \sum_{i=0}^{N-1}\Phi_{N,i}\overline{\Omega}_i A^{-1} B\, u_{k_N+i}.$$

Since $\Omega_N > 0$, pre-multiplying both sides by Ω_N^{-1} and defining filter gains $H_{N,i}, L_{N,i}$ as

$$H_{N,i} = \Omega_N^{-1}\Phi_{N,i} C^T R^{-1},\ L_{N,i} = \Omega_N^{-1}\Phi_{N,i}\overline{\Omega}_i A^{-1} B$$

yields a standard FIR form as

$$\hat{x}_{k|k} = \sum_{i=0}^{N-1} H_{N,i} y_{k_N+i} + \sum_{i=0}^{N-1} L_{N,i} u_{k_N+i}. \quad (18)$$

As shown in Fig. 2, for time-varying systems, the computation of $\Phi_{k,i}$ is repeated for all horizons. However, for time-invariant systems, since Ω_i is shift invariant, $\Phi_{N,i}$ is determined only on the interval $[0, N]$ uniquely. This means that filter gains $H_{N,i}, L_{N,i}$ require computation only on the interval $[0, N]$ once and are time-invariant for all horizons.

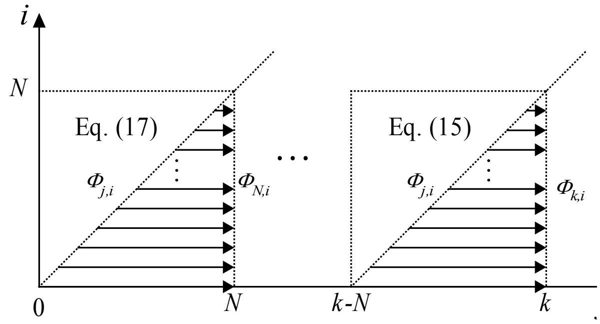

Fig. 2. Computation procedure $\Phi_{k,i}$ and $\Phi_{N,i}$.

The standard FIR form of the suggested filter differs in several respects from existing optimal FIR filters [1][2]. It provides a predicted estimate while existing filters provide a filtered estimate. It contains external control inputs unlike existing ones. Since the suggested RH FIR filter is derived from the Kalman filter, its standard FIR form provides simpler algorithms in obtaining filter gains $H_{k,i}, L_{k,i}$ or $H_{N,i}, L_{N,i}$ than existing ones. Thus, the computational burden of the suggested filter is reduced in comparison to existing optimal FIR filters.

IV. Properties of RH FIR filter

In this section, it will be shown that the RH FIR filter has the deadbeat property and the unbiasedness property. In the following theorem, it will be shown that the RH FIR filter becomes a remarkable deadbeat observer when applied to noise-free systems.

Theorem 3: Assume that $\{A_k, C_k\}$ is uniformly completely observable. Then RH FIR filters (10) and (16) for discrete time-varying systems are exact for noise-free systems for for any $N \geq l_0$.

Proof: Consider a linear discrete time-varying state-space model (1) and (2) when there is no noise as

$$x_{k+1} = A_k x_k + B_k u_k,\ y_k = C_k x_k .$$

Since $\Omega_{k_N|k} = 0$, $\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} = \Omega_{k_N+i|k}x_{k_N+i|k} = 0$ holds for $i=0$ irrespective of $\hat{x}_{k_N|k}$ and $x_{k_N|k}$. Assume that $\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} = \Omega_{k_N+i|k}x_{k_N+i|k}$ holds for i. Then, from (9), we can show that $\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} = \Omega_{k_N+i|k}x_{k_N+i|k}$ holds for $i+1$ as follows:

$$\begin{aligned}
&\Omega_{k_N+i+1|k}\hat{x}_{k_N+i+1|k} \\
&=[I + A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k}A_{k_N+i}^{-1}G_{k_N+i}Q_{k_N+i}G_{k_N+i}^T]^{-1}A_{k_N+i}^{-T} \\
&\quad[\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} + C_{k_N+i}^T R_{k_N+i}^{-1}y_{k_N+i} + \overline{\Omega}_{k_N+i|k}A_{k_N+i}^{-1}B_{k_N+i}u_{k_N+i}] \\
&=[I + A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k}A_{k_N+i}^{-1}G_{k_N+i}Q_{k_N+i}G_{k_N+i}^T]^{-1}A_{k_N+i}^{-T}\overline{\Omega}_{k_N+i|k}A_{k_N+i}^{-1} \\
&\quad[A_{k_N+i}\hat{x}_{k_N+i|k} + B_{k_N+i}u_{k_N+i}] \\
&=\Omega_{k_N+i+1|k}x_{k_N+i+1|k}.
\end{aligned}$$

Therefore, $\Omega_{k_N+i|k}\hat{x}_{k_N+i|k} = \Omega_{k_N+i|k}x_{k_N+i|k}$ holds for all $i \geq 0$. Since $\Omega_{k|k} > 0$ at the present time k, $\hat{x}_{k|k} = x_{k|k}$. This completes the proof. ■

It is noted that the RH FIR filter for time-invariant systems has the deadbeat property for any $N \geq n$ [6].

This deadbeat property indicates the finite convergence time and the fast tracking ability of the RH FIR filter. Thus, we can expect that the suggested filter would be appropriate for quick estimation and detection of signals with unknown times of occurrence, which arise in many areas such as fault detection and diagnosis of various systems, maneuver detection and target tracking of flying objects, etc. It is noted that the suggested RH FIR filter can be used as a very special deadbeat observer for noise-free systems. In this case, it is believed that this deadbeat observer is often more robust against system and measurement noises than existing ones [12][13] which did not consider the effect of these noises.

In the following theorem, it will be shown that the RH FIR filter with unknown horizon initial state is an unbiased estimator irrespective of any horizon initial condition.

Theorem 4: Assume that $\{A_k, C_k\}$ is uniformly completely observable. Then RH FIR filters (10) and (16) for discrete time-varying systems are unbiased for any $N \geq l_0$.

Proof: This is proved directly from Theorem 3. ■

It is noted that the RH FIR filter for time-invariant systems has the unbiasedness property for any $N \geq n$ [6].

V. Square-root algorithm of RH FIR filter

So far we have derived the iterative form (10) and the standard FIR form (16) of the suggested RH FIR filter for discrete time-varying systems. It can be seen that the standard FIR form of the RH FIR filter for large N requires a large number of multiplications and a large memory for filter gains. That is, the iterative form has advantages in computational burden and memory requirement compared with the standard FIR form. For this iterative form (10) of the RH FIR filter, we will present a square-root algorithm for easier parallel and systolic implementation as well as more reliable computation .

For convenience we first introduce some notational conventions. When a positive definite matrix X is given, a square-root factor $X^{1/2}$ will be defined as any matrix in a such way that $X = (X^{1/2})(X^{1/2})^T$. In most applications, such square-root factors can be made unique by insisting that they be triangular. For convenience we shall also write $(X^{1/2})^T = X^{T/2}$, $(X^{1/2})^{-1} = X^{-1/2}$, $(X^{-1/2})^T = X^{-T/2}$. Thus, let us note the expression $X = X^{1/2}X^{T/2}$, $X^{-1} = X^{-1/2}X^{-T/2}$. We also assume that a unitary operator Θ is applied to the X so as to get some special form of a matrix Y such as $X\Theta = Y$, then we shall call the X a pre-array and the Y a post-array.

The matrices $\Omega_{k_N+i|k}$ propagated by the discrete time-varying Riccati equation (8) can lose their theoretically required positive-definiteness because of the accumulation of numerical errors. In some situation, even the diagonal entries of $\Omega_{k_N+i|k}$ may become negative, resulting in absolutely meaningless state estimates. To avoid such circumstances, it is widely recommended to propagated square-root factors, $\Omega_{k_N+i|k}^{1/2}$. While numerical effects will still be present, $(\Omega_{k_N+i|k}^{1/2})(\Omega_{k_N+i|k}^{T/2})$ is much more likely to lead to a positive definite matrix since in fact the diagonal elements of the product will now always be positive. Therefore, a square-root algorithm provides the numerical stability that improves computational reliability. It is also clear that the computation of the state estimate $\hat{x}_{k|k}$ consists mainly of the time-consuming computation of $\Omega_{k_N+i|k}$. In this case, the propagation of square-root factors $\Omega_{k_N+i|k}^{1/2}$ has the advantage of the amenability to easier parallel and systolic implementation that overcomes computational burden. It is noted before that the RH FIR filter is often robust against numerical errors due to its FIR structure. Therefore, we can mention that the square-root algorithm for the RH FIR filter is more needed for the amenability to parallel and systolic implementation than for the numerical stability.

We now present a square-root algorithm for the RH FIR filter by combining the information form square-root algorithm [9] for the Kalman filter with the receding horizon strategy. We define $\hat{\overline{\eta}}_{k_N+i|k} \equiv \Omega_{k_N+i|k}^{1/2}\hat{x}_{k_N+i|k}$. Applying inner- and cross-products of the array rows, we can establish the square-root algorithm for the RH FIR filter (10) with control inputs on the horizon $[k_N, k]$ as

$$\left[\begin{array}{c}
-A_{k_N+i}^{-T}C_{k_N+i}^T R_{k_N+i}^{-T/2} \\
Q_{k_N+i}^{1/T}G_{k_N+i}^T A_{k_N+i}^{-T}C_{k_N+i}^T R_{k_N+i}^{-T/2} \\
\hline
-(y_{k_N+i}^T + u_{k_N+i}^T B_{k_N+i}^T A_{k_N+i}^{-T}C_{k_N+i}^T)R_{k_N+i}^{-T/2}
\end{array}\right.$$

$$\left[\begin{array}{cc} -A_{k_N+i}^{-T}\Omega_{k_N+i|k}^{T/2} & 0 \\ -Q_{k_N+i}^{T/2}G_{k_N+i}^{T}A_{k_N+i}^{-T}\Omega_{k_N+i|k}^{T/2} & I \\ \hline u_{k_N+i}^{T}B_{k_N+i}^{T}A_{k_N+i}^{-T}\Omega_{k_N+i|k}^{T/2}+\hat{\bar{\eta}}_{k_N+i|k}^{T} & 0 \end{array}\right]\times\Theta_{k_N+i}$$

$$=\left[\begin{array}{ccc} 0 & \Omega_{k_N+i|k}^{T/2} & * \\ 0 & 0 & * \\ \hline * & \hat{\bar{\eta}}_{k_N+i|k}^{T} & * \end{array}\right],\quad 0\le i\le N-1 \tag{19}$$

with the horizon initial condition $\hat{\bar{\eta}}_{k_N|k}=\Omega_{k_N|k}^{1/2}\hat{x}_{k_N|k}=0$. It is noted that " $*$ " indicates a redundant entry and Θ_{k_N+i}, is any orthogonal rotation, $\Theta_{k_N+i}\Theta_{k_N+i}^{T}=I$, that upper-triangularizes the first two rows of the pre-array.

It can be seen from (19) that an intermediate variable to compute the real state estimate $\hat{x}_{k|k}$ can be found from the entries of the post array by solving the triangular system $\hat{\bar{\eta}}_{k_N+i+1|k}$. At the present time k , we can obtain the real state estimate $\hat{x}_{k|k}=\Omega_{k_N+i|k}^{-1/2}\hat{\bar{\eta}}_{k|k}$ where the available value is given. Therefore, it can be stated that the square-root algorithm for the RH FIR filter is amenable to parallel and systolic implementation since the state estimate is found as products of quantities that are available from the post-array.

VI. Application to unknown input estimation

It was mentioned previously that the RH FIR filter has a deadbeat property, which means the finite convergence and the quick tracking ability of the RH FIR filter. It is also known that the increase of the number of observations for a detection decision will increase the detection delay in detecting a signal with unknown time of occurrence [14]. Therefore, it may be expected that the unknown input estimation using the RH FIR filter can provide quicker estimation than the approach using IIR filter such as the Kalman filter that doesn't provide a deadbeat property.

Therefore, in this section, the RH FIR filter based unknown input estimation and the Kalman filter based Friedland's approach in [15] are compared. Two approaches are applied to the problem of the following DC motor system:

$$x_{k+1}=\begin{bmatrix}-0.0005 & -0.0084\\ 0.0517 & 0.8069\end{bmatrix}x_k+\begin{bmatrix}0.1815\\ 1.7902\end{bmatrix}u_k +\begin{bmatrix}0.0129 & 0\\ -1.2504 & 0\end{bmatrix}p_k+\begin{bmatrix}0.0006\\ 0.0057\end{bmatrix}v_k, \tag{20}$$

$$y_k=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x_k+\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}p_k+v_k. \tag{21}$$

When the unknown inputs $p_k=[p_k^1\ \ p_k^2]^T=p_{k-1}+\delta_{k-1}$ with a random-walk type are treated as auxiliary states, the system (20) and (21) can be augmented as (1) and (2) of the fourth order state space model with state variable $x_k=[x_k^T\ \ p_k^T]^T$ and system noise $w_k=[w_k^T\ \ \delta_k^T]^T$. Then, for the augmented system, the RH FIR filter based approach and the Kalman filter based Friedland's approach in [15] are implemented for the unknown input estimation.

In the simulation, the first unknown input p_k^1 is modeled as step function that changes at $k=100$ with the magnitude of 0.5 and the second one p_k^2 is modeled as zero. The horizon length is taken as $N=10$. As shown in Fig. 3 and 4, the RH FIR filter based approach has a quicker estimation performance than the Friedland's approach at time of unknown input occurrence. In addition, in the RH FIR filter based approach, the first constant unknown input does not affect the estimate of the second unknown input. However, in the Friedland's approach, the first constant unknown input affects the estimate of the second unknown input.

Besides this application, the RH FIR filter for time-varying systems might be useful for various applications which require time-varying system. In time-invariant systems, the RH FIR filter has turn out to be a useful practice for solving problems of filter divergence due to modeling uncertainty [6], for estimating signal with quasi-periodic components [16], etc.

VII. Conclusion

Some contributions of the current work can be briefly summarized as follows. The derivation of the suggested RH FIR filter is easier to understand than previous results since it comes from a modification of the well known Kalman filter. The iterative form of the suggested filter can always be obtained irrespective of singularity problems caused by unknown information about the horizon initial state. It has been shown

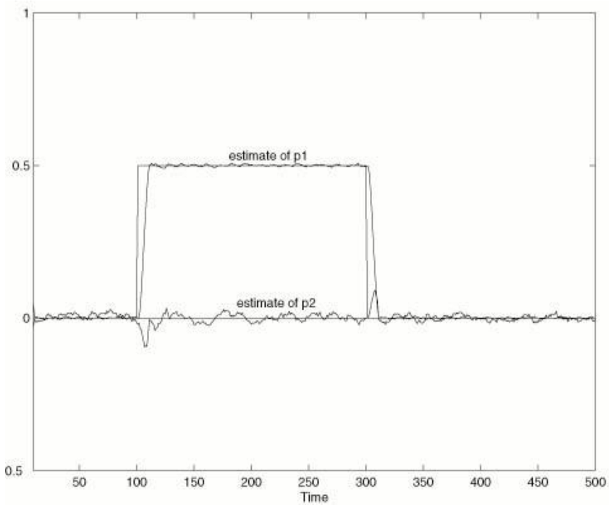

Fig. 3. Result of RH FIR filter based approach.

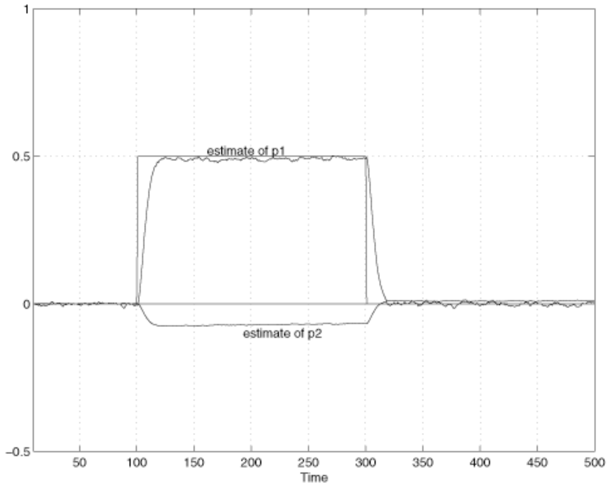

Fig. 4. Result of friedland's approach.

that the suggested iterative filter can be represented in a standard FIR form, which provides simpler algorithms for obtaining filter gains than existing optimal FIR filters. The suggested filter includes a control input term and thus can be applied to feedback control problems. As a by-product, we obtain a remarkable deadbeat observer, which indicates the finite convergence time and the fast tracking ability of the suggested filter. The square-root algorithm for the suggested filter will provide many advantages with respect to the amenability to parallel and systolic implementation as well as with respect to the numerical stability.

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Pyung Soo Kim

Pyung Soo Kim was born in Korea on February 5, 1972. He received the B.S. degree in electrical engineering from Inha University, Inchon, Korea in 1994 and the M.S. degree in control and instrumentation engineering from Seoul National University, Seoul, Korea in 1996. He is currently a Ph. D. candidate in the School of Electrical Engineering, Seoul National University. His main research interests are in the areas of statistical signal processing, fault detection and identification, and industrial application.

Wook Hyun Kwon

Wook Hyun Kwon was born in Korea on January 19, 1943. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1966 and 1972, respectively. He received the Ph.D. degree from Brown University, Providence, RI, in 1975. Since 1977, he has been with the School of Electrical Engineering, Seoul National University. His main research interests are currently multivariable robust and predictive controls, statistical signal processing, discrete event systems, and industrial networks.