

# Control Education Using Pendulum Apparatus

Tasuku Hoshino, Masaki Yamakita, and Katsuhisa Furuta

**Abstract:** The inverted pendulum is a typical example of unstable systems and has been used for verification of designed control systems. It is also very popular in control education in laboratories, serving as a good example to show the utility of the state space approach to the controller design. This paper shows two kinds of experiment using inverted pendulum: one is the stabilization of a single spherical inverted pendulum by a plane manipulator using visual feedback, and the other is the state transfer control of a double pendulum. In the former experiment, the feedback stabilization using a CCD camera has major importance as an example of controller implementation with non-contact measurement. The latter involves the standard stabilizing regulation method and nonlinear control techniques. The details of the experimental systems, the control algorithms and the experimental results will be given.

**Keywords:** control education, inverted pendulum, visual feedback, nonlinear control

## I. Introduction

The inverted pendulum has been widely used in control laboratories to demonstrate the effectiveness of feedback control in analogy with the control of launching a rocket [1], [7], [4]. Stabilization of not only a single inverted pendulum but also a double or a triple inverted pendulum have been studied by many researchers [5], [6], [8], [3]. Because of its simple structure, tractable nonlinearity and the clear design goal, it works as a good example for controller design in control education.

When considering about implementation of feedback systems, non-contact measurement is a very important technique. Especially in robotics, image processing is essential for the guidance of mobile robots. In order to jointly demonstrate the image processing and stabilization of a system, we investigate a system which consists of a SCARA manipulator, CCD camera and a spherical pendulum. This system implements visual stabilizing feedback control of the pendulum using its camera image. The stabilization is just like as that a person is stabilizing an umbrella on his palm. Using the system, students easily understand the image processing, mainly the field of perspective projection and position reconstruction, and synthesis of stabilizing feedback control.

The stabilization of systems around equilibrium point, just as described above, is one objective of control; however, there exists another important problem: to transfer a system state from one equilibrium to another. Transferring a state of a pendulum from pendent to inverted may be the most typical example of such a problem [1], [12] and called the swing-up control. In this case, the stabilizing control may be used in its final stage in the neighborhood of the equilibrium point. The state transfer control of a double pendulum is an extension of the problem in [1]. In the double pendulum, there are four equilibrium states namely Down-Down, Down-Up, Up-Down and Up-Up states

which will be defined later. A variety of control methods are possible when transferring the state from one equilibrium to the others in the context of nonlinear control system design.

In this paper, the authors deal with two experimental systems highlighting the two control problems above: the stabilization of the single spherical inverted pendulum with a planer manipulator using visual feedback and the state transfer control of a double inverted pendulum (referred to as TITech or Furuta pendulum). These two practical examples involve additional features for undergraduate or advanced control education, comparing with the conventional experimental systems using inverted pendulum. The following sections describe each system in order.

## II. Visual feedback control of a spherical pendulum

### 1. System description

The single spherical pendulum system is shown in Figure 1. It consists of a solid aluminum rod and a 2 d.o.f. SCARA manipulator and can be regarded as a serial connection of three rigid links. The SCARA manipulator is driven by two DD motors. The kinematics is systematically solved by the use of homogeneous transformations between coordinate systems which are attached to each link in Denavit-Hartenberg manner [15]. The coordinate systems in our case are shown in Figure 2. The total kinetic energy  $T$ , the total potential energy  $U$  and the total dissipative energy  $R$  determine the equations of motion through Lagrangian formulation

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i - \frac{\partial R}{\partial \dot{q}_i}, \quad i \in \{1, 2, 3, 4\}$$

Fig. 1. A single spherical pendulum.

where  $L = T - U$  is Lagrangian,  $q_i$ 's are generalized coordinates,  $Q_i$ 's are generalized forces. Since all the joints are revolute,  $q_i$ 's are joint angles  $\theta_i (i \in \{1, 2, 3, 4\})$ , and since only the two joints of SCARA manipulator are actuated,  $Q_i$ 's are joint torques  $\tau_i (i \in \{1, 2\})$  and zero otherwise.

Manuscript received: May 15, 2000., Accepted: Sept. 8, 2000.  
*Tasuku Hoshino:* Dept. of Management and Systems Science,  
 The Science University of Tokyo

*Masaki Yamakita:* Dept. of Mechanical and Control Systems  
 Engineering, Tokyo Institute of Technology

*Katsuhisa Furuta:* Dept. of Computers and Systems Engineering,  
 Tokyo Denki University

The authors are grateful to reviewers for their careful reading and valuable comments and suggestions.

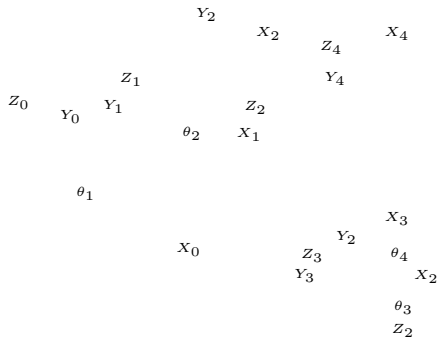


Fig. 2. Coordinate frames.

Lagrange's equations give four equations of motions:

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{V}\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}) = \boldsymbol{\tau}, \quad (1)$$

where  $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \theta_4]^T$  and  $\boldsymbol{\tau} = [\tau_1, \tau_2, 0, 0]^T$ ;  $\mathbf{M}(\cdot)$ ,  $\mathbf{V}$  and  $\mathbf{C}(\cdot, \cdot)$ ,  $\mathbf{G}(\cdot)$  are matrices and vectors of appropriate dimensions. The entries are complicated and involving trigonometric functions. Derivation of the model was performed using a symbolic manipulation tool of equations such as *Mathematica*.

By linearly approximating equation (1) around an equilibrium point  $\boldsymbol{\theta}_0 = [0, \pi/2, \pi/2, 0]^T$ , the equations of motion for the 2-D pendulum with SCARA manipulator can be represented in a linear state equation:

Table 1. DH parameters for 2-D pendulum

$i$	$a_i$	$d_i$	$\alpha_i$	$\theta_i$
1	$a_1$	0	0	$\theta_1$
2	$a_2$	0	$\pi/2$	$\theta_2$
3	$\varepsilon (=0)$	0	$-\pi/2$	$\theta_3$
4	$l$	0	0	$\theta_4$

Table 2. Model parameters

$g$	the gravity acceleration
$a_1$	length of 1st manipulator link
$a_2$	length of 2nd manipulator link
$l_p$	a half of length of the pendulum ( $= l/2$ )
$m_1$	mass of 1st link
$m_2$	mass of 2nd link
$m_p$	mass of the pendulum
$J_1$	inertia of 1st link around the joint
$J_2$	inertia of 2nd link around the joint
$J_p$	inertia of the pendulum around the bottom
$V_1$	friction coefficient of 1st joint
$V_2$	friction coefficient of 2nd joint

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{O}_4 & \mathbf{I}_4 \\ -\mathbf{M}_0^{-1}\mathbf{G}_0 & -\mathbf{M}_0^{-1}\mathbf{V}_0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{O}_4 \\ \mathbf{M}_0^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{O}_2 \end{bmatrix} \mathbf{u} \quad (2)$$

where  $\mathbf{x} := \boldsymbol{\theta} - \boldsymbol{\theta}_0$ ,  $\mathbf{u} := [\tau_1, \tau_2]^T$ , and

$$\mathbf{M}_0 = \begin{bmatrix} J_1 + J_2 + m_2 a_1^2 & J_2 + m_p a_2^2 & -m_p a_1 l_p & m_p a_2 l_p \\ +m_p a_1^2 + m_p a_2^2 & J_2 + m_p a_2^2 & 0 & m_p a_2 l_p \\ -m_p a_1 l_p & 0 & J_p & 0 \\ m_p a_2 l_p & m_p a_2 l_p & 0 & J_p \end{bmatrix},$$

$$\mathbf{V}_0 = \begin{bmatrix} V_1 & 0 & \mathbf{O}_2 \\ 0 & V_2 & \mathbf{O}_2 \end{bmatrix}, \quad \mathbf{G}_0 = \begin{bmatrix} \mathbf{O}_2 & -m_p l_p g & \mathbf{O}_2 & 0 \\ \mathbf{O}_2 & 0 & -m_p l_p g & \mathbf{O}_2 \end{bmatrix}.$$

$\mathbf{M}_0, \mathbf{V}_0, \mathbf{G}_0$  are an inertia matrix, friction term and gravity term respectively, linearly approximated at the equilibrium state.

## 2. Visual measuring

Taking two joint angles of SCARA and the position of the top of the pendulum  $[x_t, y_t]^T$  as outputs, the linearly approximated measurement equation is given as follows:

$$\mathbf{y} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a_2 & -a_2 & 0 & -l \\ a_1 & 0 & 0 & -l \end{bmatrix} \mathbf{O}_4 \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}. \quad (3)$$

The top position is visually measured by a CCD camera. In general, the CCD camera can be modeled as a perspective projection from the workspace coordinate  $R^3 \ni [x, y, z]^T$  to the camera image coordinate  $[u, v]^T \in R^2$ . To determine the workspace coordinate from the measurement, stereo vision is required. However, since the angle deviation of the pendulum is small, the top height is nearly fixed; therefore 2-D measurement is enough for the purpose. Moreover, the camera model is reduced by assuming that the workspace coordinate of the top  $[x_t, y_t]^T$  is given by its camera coordinate  $[u_t, v_t]^T$  in the following affine relation:

$$\begin{aligned} & \text{camera coordinates: } (u_t, v_t)^T \\ & \text{focal point} \\ & \text{workspace coordinates: } (x_t, y_t)^T \\ & \text{pendulum} \end{aligned}$$

Fig. 3. Measuring with a CCD camera.

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} k_{x0} \\ k_{y0} \end{bmatrix} + \begin{bmatrix} k_{xu} & k_{xv} \\ k_{yu} & k_{yv} \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix}. \quad (4)$$

This makes the calibration process easier: the parameters  $k_{x0}, k_{xu}, k_{xv}, k_{y0}, k_{yu}, k_{yv}$  are determined by minimizing the residuals

$$\begin{aligned} S_{xx} &= \sum_{i=1}^{25} \{x_{ti} - (k_{x0} + k_{xu}u_{ti} + k_{xv}v_{ti})\}^2, \\ S_{yy} &= \sum_{i=1}^{25} \{y_{ti} - (k_{y0} + k_{yu}u_{ti} + k_{yv}v_{ti})\}^2, \end{aligned}$$

which can be solved by the linear least square method. Equation (4) is quite simple but it yields enough estimates of top position of the pendulum as shown in Figure 4.

## 3. Experiments

Several controller design methods are applicable for system  $\{(2), (3)\}$ . For our educational purpose, a LQ regulator with state observer was chosen. Figure 5 is a block diagram of the overall system. The experimental result is shown in Figure 6.

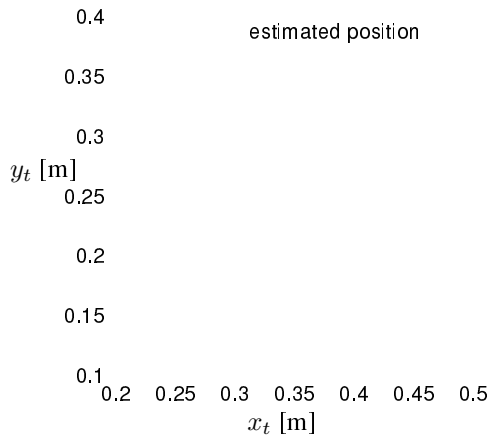


Fig. 4. Calibration result.

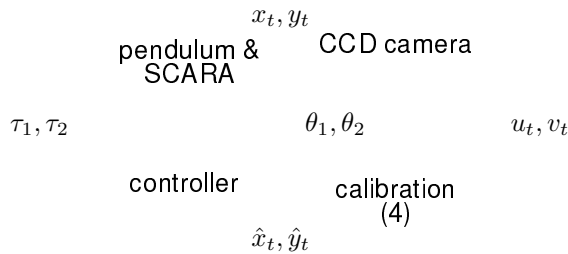


Fig. 5. Closed loop system.

Fig. 6. Angle deviations from up-right position.

$x_3$  and  $x_4$  stand for  $x_3$  and  $x_4$ , the 3rd and 4th entries of  $\mathbf{x}$ , which represent the angle deviations of the pendulum in two directions from its upright position. While there appears low-frequency persistent oscillation in the angle deviations due to the dry friction at the manipulator joints, the figure shows that the stabilization in both directions was successfully carried out.

### III. State transfer control of a double pendulum

#### 1. System description

We developed a rotationally driven double pendulum (the TITech double pendulum) shown in Figures 7 and 8. It has following advantages against the standard pendulum on a linear rail:

1. The first joint around which control input is exerted does not have hardware limitation on the movement since it is rotational joint.
2. The first pendulum is attached directly to a rigid arm fixed to the rotor of the actuator; there exists less dynamic uncertainties than conventional cart type systems.
3. The actuator is a DD motor. It is free from backlash and flexibility of gear transmissions.

The first characteristic is essential in order to implement the state transfer control, since especially in the double pendulum case, long traveling distance of link 0 around the first joint is necessary. Details of the modeling can be found in [13], [14].

Fig. 7. The TITech double pendulum.

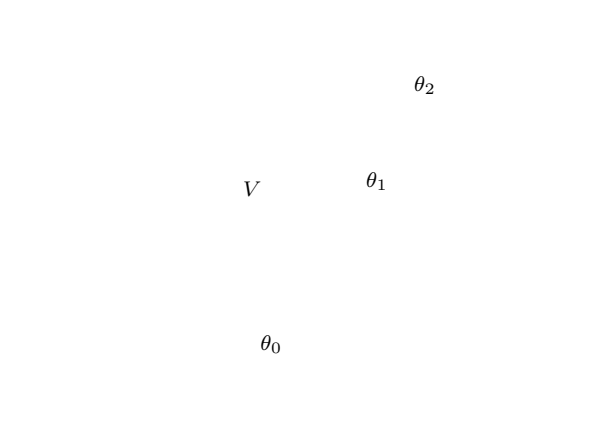


Fig. 8. TITech double pendulum and its coordinate systems.

#### 2. State transfer control from one equilibrium to others

In the TITech double pendulum, there are four equilibrium states as shown in Figure 9. Moreover, there is relative equilibrium with limit cycle where link 1 is standing and link 2 is rotating. Stabilization of these equilibria itself is of interest study, but the authors proceed to transferring state from one of the equilibria to another. There are several paths for the state transfer as shown in Figure 10. Once state transfers represented by these paths are established by controlling each, the swing up control will be made possible by connecting appropriate paths.

In the following, the authors firstly consider the state transfer control of path 11, 9, 16 and 13. Then by connecting these paths, swing-up of the double pendulum by switching control is constructed. This is a flexible approach to derive a switching control law for the swinging up control.

##### 2.1 State transfer control from down-down to up-down

For the state transfer control, we employ a method proposed in [12] based on energy control. when we apply the control method to our system, we ignore the effects of link 2 as disturbances since the weight of link 2 is rather less than that of link 1 in our system. The basic idea is as follows. Let  $U_1$  be total energy of link 1 consisting of potential and kinematic energy. Taking  $U_1 - U_{1d}$  as a criterion function where  $U_{1d}$  is a total energy corresponding to the Up-Down state, the control input is determined so that the criterion function approaches to zero. Since  $\theta_0$  will become unstable during the control, a stabilizing control for  $\theta_0$  is applied after the Up-Down state is achieved. See the details in [12].

##### 2.2 State transfer control from Up-Up to Up-Down

The basic strategy of the state transfer control from Up-Up to Up-Down is to robustly stabilize link 1 against disturbance caused by the motion of link 2, during the state is far from the target Up-Down state. This is possible because the stabilization of a single inverted pendulum is very robust against external force disturbances applied to the pendulum rod. In our system we employ a quadratic stabilization method as a robust control algorithm as in [14].

##### 2.3 Stabilizing a limit cycle

Fig. 9. Four equilibrium states of the system.

Fig. 10. Paths from one equilibrium to another.

Fig. 11. State transition (path 11 and 9).

Fig. 12. State transition (path 11, 16 and 13).

When the authors repeatedly performed the experiments of the state transfer control from Up-Up to Up-Down, a kind of pendulum motion, like a limit cycle, was found;  $\theta_0$  and  $\theta_1$  are maintained in the neighborhood of Up-Down state and  $\theta_2$  is rotating. Furthermore, the period of the rotation proved to be possible to control by a scalar controller parameter. The mechanism of the control has not been fully analyzed yet; but the phenomenon appears every time. We concluded that the control method is very robust for the initial conditions from the observing experiments of several times. Note that in this case  $\dot{\theta}_2$  is not taken into account in measuring the distance from the Up-Down state. State transfer control from the limit cycle to Up-Up (path 15) and from the limit cycle to Up-Down (path 14) and vice versa can be easily implemented by choosing the control gain. Consult [14] for more details.

#### 2.4 State transfer control from Up-Down to Up-Up

The basic idea is similar to that of the state transfer control from Down-Down to Up-Down. It must be considered, however, to maintain  $\theta_0$  and  $\theta_1$  in the neighborhood of the Up-Down in this case. Let a partial energy of link 2 be  $U_2$ :

$$U_2 = \frac{1}{2}(J_2 + m_2 l_2^2) \dot{\theta}_2^2 + m_2 g l_2 (1 + \cos \theta_2),$$

where  $J_2, m_2, l_2$  is the inertial moment, the mass and the position of link 2, respectively. Taking the following function as a criterion function,

$$V = U_2 - U_{2d}$$

where

$$U_{2d} = 2m_2 g l_2 + \alpha, \quad \alpha > 0.$$

We can derive a control input by taking a derivative of the above criterion function as in [14]. Unfortunately, this control law is not available when  $\dot{\theta}_2 = 0$  and, if we continuously use this control law,  $\theta_1$  will become unstable. Therefore we modified the actual control method as follows consisting two phases:  
*Step1* If the state  $(\theta_1, \theta_2)$  are in the neighborhood of Up-Down ones, the above control law is used to control  $U_2$ .  
*Step2* If  $(\theta_1, \theta_2)$  are far from the Up-Down state, the control method in the state transfer control from Up-Up to Up-Down is applied.

### 3. Experimental results

Figure 11 shows one of the experimental results, where the state transfer from Down-Down to Up-Up via Up-Down (path 11 and 9 in Figure 10). Figure 12 shows another state transfer from Down-Down to Up-Up through the limit cycle (path 11, 16 and 13 in Figure 10). In both cases, the state was successfully transferred one after another.

## IV. Conclusions

In this paper we have shown two experimental systems using inverted pendulum for control education. We have been using a single inverted pendulum system over 10 years as an experimental system in undergraduate experimental course. Since

students get very excited to stabilize the system in the class, we think that they will also be interested in the systems shown in this paper. We believe that this paper will give a cure to develop good experimental systems for control education.

## References

- [1] S. Mori, H. Nishihara, and K. Furuta, "Control of unstable mechanical system Control of pendulum," *Int. J. Control*, vol. 23, pp. 673-692, 1976.

- [2] N. Bakri, N. Becker, and E. Ostertag, "Application of control disturbance observers for the control and compensation of dry friction," *Automatisierungstechnik*, vol. 36, pp. 50-54, 1988.
- [3] W. R. Sturgeon and M. V. Loscutoff, "Application of modal control and dynamic observers to control of a double inverted pendulum," in *Proc. JACC*, Stanford, pp. 857-865, 1972.
- [4] K. Furuta, H. Kajiwara, and K. Kosuge, "Digital control of a double inverted pendulum on an inclined rail," *Int. J. Control*, vol. 32, pp. 907-924, 1980.
- [5] K. Furuta, T. Ochiai, and N. Ono, "Attitude control of a triple inverted of pendulum," *Int. J. Control*, vol. 39, pp. 1351-1365, 1984.
- [6] W. Maletinsky, M. F. Senning, and F. Wiederkehr, "Observer based control of a double pendulum," *Proc. IFAC World Congr.*, pp. 3383-3387, 1981.
- [7] K. Furuta, A. Sano, and D. Atherton, *State variable methods in automatic control*, WILEY, 1988.
- [8] H. Meier, Zu Farwig, and H. Unbehauen, "Discrete computer control of a triple-inverted pendulum," *Opt. Cont. App. & Methods*, vol. 11, pp. 157-171, 1990.
- [9] R. Luus, "Optimal control by dynamic programming using systematic reduction in grid size," *Int. J. of Control*, vol. 51, no. 5, pp. 995-1023, 1990.
- [10] M. Yamakita and K. Furuta, "Iterative generation of virtual reference for a manipulator," *Robotica*, vol. 9, pp. 71-80, 1991.
- [11] K. Furuta, M. Yamakita, and S. Kobayashi, "Swing up control of inverted pendulum using pseudo-state feedback," *J. Systems and Control Engineering*, vol. 206, pp. 263-269, 1992.
- [12] M. Wiklund, A. Kristenson, and K. J. Astrom, "A new strategy for swinging up an inverted pendulum," in *Proc. of IFAC Congress*, vol. 9, pp. 151-154, 1992.
- [13] M. Yamakita, K. Nonaka, and K. Furuta, "Swing up control of double pendulum," in *Proc. of ACC'93*, pp. 2229-2234, 1993.
- [14] M. Yamakita, M. Iwashiro, Y. Sugahara, and K. Furuta, "Robust swing up control double pendulum," in *Proc. of ACC'95*, pp. 290-296, 1995.
- [15] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, Inc., 1989.
- [16] T. Hoshino and K. Furuta, "Stabilization of 2-D inverted pendulum via partial exact linearization," in *Proc. of ASSC'97*, pp. 495-498, 1997.

#### Tasuku Hoshino

He received his B.E. in Electrical Engineering from the Science University of Tokyo in 1990, and his M.E. and Ph.D. degrees in Control Engineering from the Tokyo Institute of Technology in 1992 and 2000. Since 1992 to 2000, he was a research associate at the Tokyo Institute of Technology. His research interests include nonlinear control of mechanical systems, robotics, and their implementation. He is a member of SICE, JRS and IEEE.

#### Katsuhisa Furuta

He was born in Tokyo, Japan in 1940. He received his B.S., M.S., and Ph.D. degrees in Engineering from the Tokyo Institute of Technology in 1962, 1964, and 1967, respectively. He was a post doctoral fellow at Laval University (Quebec, Canada) from July, 1967 to August, 1969. Since then, he has been a member of the teaching staff of the Tokyo Institute of Technology, Department of Control Engineering, where he is currently a Professor, Graduate School of Information Science and Engineering. He was a Russell Severance Springer Professor, University of California at Berkeley in 1997. He is a member of Science Council of Japan from 1997, and is the President of SICE (Society of Instrument and Control Engineers) in 1999. He has been a council member of IFAC and the editor of *Automatica* in application from 1996 till 1999. He is a fellow of both IEEE and SICE and has received the honorary doctor from Helsinki University of Technology.

#### Masaki Yamakita

He was born in 1962. He graduated in 1985, received M.S.E. in 1986 and Ph.D. in 1989 from Tokyo Institute of Technology. He was an associate researcher from 1989 at the institute, and he was a lecturer at Toyohashi University of Technology from 1993. He is currently an associate professor in the department of control and Systems engineering of Tokyo Institute of Technology. His research interests include robust control, adaptive and learning control and robotics. He is a member of SICE, JRS and IEEE.