Development of Continuous/Discrete Mixed H_2/H_∞ Filtering Design Algorithms for Time Delay Systems

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Abstract: The problems of mixed H_2/H_∞ filtering design for continuous and discrete time linear systems with time delay are investigated. The main purpose is to design a stable mixed H_2/H_∞ filter which minimizes the H_2 performance measure satisfying a prescribed H_∞ norm bound on the closed loop system in continuous-time case and discrete-time case, respectively. The sufficient conditions of existence of filter, the mixed H_2/H_∞ filter design method, and the upper bound of performance measure are proposed by LMI(linear matrix inequality) techniques in terms of all finding variables. Also, we present optimization problems in order to get the optimal mixed H_2/H_∞ filter in continuous and discrete time case, respectively.

Keywords: mixed H_2/H_∞ filter, time delay systems, linear matrix inequality

I. Introduction

During the last decades, the extensive use of optimization criteria like the H_2 and/or H_∞ norm has consolidated the importance of estimation and filtering in linear systems theory. The theory of filtering has long been one of the cornerstones of modern system theory. Also, it is well known that for white noise inputs the maximal output variance leads to the H_2 criterion, while energy bounded signals corrupting the system appears to be more tractable in H_∞ setting. In the H_2 filtering approach, the noise characteristics are known leading to the minimization of the H_2 norm of the transfer function from the processes noise to the estimation error[1][2]. Recently, the H_∞ filtering approach has been developed from the loose assumption of boundedness of the noise variance. In this case, the performance index to be minimized being the worst case H_∞ norm from the process noise to the estimation error[3][4]. Recently, Geromel *et al*.[5][6] presented robust filtering design methods in H_2 space and H_∞ space, respectively. In order to get the robust performance, the filtering design problem dealing with both H_2 and H_∞ norm measures is necessary. Palhares *et al*.[7] proposed the problem of mixed L_2-L_∞/H_∞ performance filtering design for uncertain linear systems. Generally speaking, the mixed H_2/H_∞ filtering design can be described as the problem of minimizing an upper bound to the energy-to-peak gain while a prescribed γ noise attenuation level is imposed to the H_∞ norm of the filtering error system, considering two different channels. Also, Palhares *et al*.[8] considered robust H_∞ filter design algorithm with pole constraints for discrete time systems. Wang *et al*.[9] dealt with the problem of robust H_2/H_∞ state estimation for discrete time systems with error variance constraints. However, most of filtering papers did not consider time delay. More recently, there are many papers considering time delay systems in control part[10][11][12] because the time delay is frequently a source of instability and encountered in various engineering systems. However, there are no papers considering mixed H_2/H_∞ filtering design methods of time delay systems. Also, the existing works without considering time delay were much conservative because they did not give optimization methods to get an optimal filter which minimized the H_2 performance measure satisfying prescribed H_∞ norm bound of the closed loop system. Therefore, our aim is to present design methods in order to get an optimal mixed H_2/H_∞ filter in continuous and discrete time systems with time delay.

In this paper, we consider the mixed H_2/H_∞ filtering design algorithms of linear time delay systems using LMI technique. Also, we present the continuous/discrete optimization problems. Since our proposed sufficient conditions are LMIs, all solutions can be obtained at the same time. Also, a numerical example is provided to demonstrate theoretical results. Here, the notation is fairly standard. $\delta x(t)$ indicates $\dot{x}(t)$ for continuous time systems and $x(t+1)$ for discrete time systems. The symbol $*$ represents the elements below the main diagonal of a symmetric block matrix. $tr(\cdot)$ denotes the trace of the matrix $(\cdot)$.

II. Mixed H_2/H_∞ filtering design

Consider a linear time delay system

$$\begin{aligned} \delta x(t) &= Ax(t) + A_d x(t-d) + Bw(t) \\ y(t) &= Cx(t) + Dw(t) \\ x(t) &= \phi_1(t), \quad -d \le t \le 0 \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $y(t) \in \mathbf{R}^r$ is the measurement output vector, $w(t) \in \mathbf{R}^m$ is the noise signal vector, $\phi_1(t)$ is an initial value function, and all matrices have proper dimensions. Time delay d is positive real number in continuous-time case and positive integer number in discrete-time case. And we assume that the system (1) is asymptotically stable. This assumption guarantees that the boundedness of the filtering error holds, since the asymptotic stability of the filtering error dynamics depends on the states of the system (1). Our aim is to design a stable linear mixed H_2/H_∞ filter described by

$$\delta \hat{x}(t) = \hat{A}\hat{x}(t) + A_d \hat{x}(t-d) + Ky(t) \tag{2}$$

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where, $\check{A}$ and K are filter variables. If we take the error state vector as follows:

$$e(t) = x(t) - \check{x}(t), \tag{3}$$

then the error dynamics is obtained

$$\begin{aligned} \dot{e}(t) &= \check{A}e(t) + (A - KC - \check{A})x(t) \\ &\quad + A_d e(t-d) + (B - KD)w(t) \\ z_1(t) &= L_1 e(t) \\ z_2(t) &= L_2 e(t) \end{aligned} \tag{4}$$

by defining the error state output as $z_i(t) = L_i e(t)$, $i = 1, 2$. Define the following augmented state vector

$$x_f(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{5}$$

such that the filtering error dynamics is given by

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + A_{df} x_f(t-d) + D_f w(t) \\ z_1(t) &= C_{1f} x_f(t) \\ z_2(t) &= C_{2f} x_f(t) \\ x_f(t) &= \phi_f(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}, \quad -d \le t \le 0 \end{aligned} \tag{6}$$

where some notations are denoted by

$$\begin{aligned} A_f &= \begin{bmatrix} A & 0 \\ A - KC - \check{A} & \check{A} \end{bmatrix}, \quad A_{df} = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, \\ D_f &= \begin{bmatrix} B \\ B - KD \end{bmatrix}, \quad C_{1f} = [\,0 \;\; L_1\,], \quad C_{2f} = [\,0 \;\; L_2\,]. \end{aligned} \tag{7}$$

Associated with mixed $H2/H\infty$ filter, we introduce the following filtering design objective:

For a given γ determine filter variables $\check{A}$ and K that achieve minimization of H2 performance measure under satisfying $H\infty$ norm bound within predetermined γ (8)

Also, we introduce H2 performance measures

$$J_1 = \begin{cases} \int_0^\infty z_1(t)^T z_1(t)\,dt \\ \sum_{t=0}^{\infty} z_1(t)^T z_1(t) \end{cases} \tag{9}$$

and $H\infty$ performance measures

$$J_2 = \begin{cases} \int_0^\infty [z_2(t)^T z_2(t) - \gamma^2 w(t)^T w(t)]\,dt \\ \sum_{t=0}^{\infty} [z_2(t)^T z_2(t) - \gamma^2 w(t)^T w(t)] \end{cases} \tag{10}$$

in continuous time case and discrete time case, respectively. Therefore, our aim is to develop the mixed $H2/H\infty$ filtering design method satisfying the objective (8). In the following, we present LMI optimization problems to get the optimal mixed $H2/H\infty$ filter satisfying (8) by LMI technique in continuous time (Theorem 1) and discrete time (Theorem 2), respectively.

Theorem 1: (Continuous time case) For a given positive constant γ, if the following optimization problem

$\min \{\alpha + tr(Q)\}$ subject to

$$i)\;\begin{bmatrix} A^T P_1 + P_1 A + S_1 & A^T P_2 - C^T M_2^T - M_1^T + S_2 & P_1 A_d & 0 \\ * & M_1^T + M_1 + L_1^T L_1 + S_3 & 0 & P_2 A_d \\ * & * & -S_1 & -S_2 \\ * & * & * & -S_3 \end{bmatrix} < 0,$$

$$ii)\;\begin{bmatrix} A^T P_1 + P_1 A + S_1 & A^T P_2 - C^T M_2^T - M_1^T + S_2 & P_1 A_d & 0 & P_1 B \\ * & M_1^T + M_1 + L_2^T L_2 + S_3 & 0 & P_2 A_d & P_2 B - M_2 D \\ * & * & -S_1 & -S_2 & 0 \\ * & * & * & -S_3 & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$

$$\begin{aligned} iii)\;& -\alpha + \phi_1(0)^T P_1 \phi_1(0) + \phi_2(0)^T P_2 \phi_2(0) < 0, \\ iv)\;& -Q + N_1^T S_1 N_1 + N_2^T S_2 N_1 + N_1^T S_2 N_2 + N_2^T S_3 N_2 < 0 \end{aligned} \tag{11}$$

has a solution positive definite matrices(or scalar) P_1, P_2, S_1, S_2, S_3, α, Q, and matrices M_1, M_2, then (2) is a continuous time mixed $H2/H\infty$ filter and $J^* = \alpha + tr(Q)$ is an upper bound of continuous time $H2$ performance measure. Here, some notations are defined as

$$\begin{aligned} M_1 &= P_2 \check{A} \\ M_2 &= P_2 K \\ \int_{-d}^{0} & \phi_f(\tau)\phi_f(\tau)^T d\tau = NN^T = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} [\, N_1^T \;\; N_2^T \,]. \end{aligned} \tag{12}$$

Proof: If we define a Lyapunov functional

$$V(x_f(t)) = x_f(t)^T P x_f(t) + \int_{t-d}^{t} x_f(\tau)^T S x_f(\tau)\,d\tau, \tag{13}$$

then the derivative of (13) is given

$$\begin{aligned} \dot{V}(x_f(t)) &= \dot{x}_f(t)^T P x_f(t) + x_f(t)^T P \dot{x}_f(t) \\ &\quad + x_f(t)^T S x_f(t) - x_f(t-d)^T S x_f(t-d). \end{aligned} \tag{14}$$

The linear matrix inequality (i) in (11) implies that

$$\dot{V}(x_f(t)) < -z_1(t)^T z_1(t) < 0. \tag{15}$$

Therefore we have

$$\begin{bmatrix} x_f(t) \\ x_f(t-d) \end{bmatrix}^T \times \begin{bmatrix} A_f^T P + PA_f + C_{1f}^T C_{1f} + S & PA_{df} \\ * & -S \end{bmatrix} \begin{bmatrix} x_f(t) \\ x_f(t-d) \end{bmatrix} < 0, \tag{16}$$

when assuming the zero noise signal vector input. And if we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix}, \tag{17}$$

then the following inequality

$$\begin{bmatrix} A_f^T P + PA_f + C_{1f}^T C_{1f} + S & PA_{df} \\ * & -S \end{bmatrix} < 0 \tag{18}$$

is changed to

$$\begin{bmatrix} A^TP_1+P_1A+S_1 & A^TP_2-C^TK^TP_2-\breve{A}^TP_2+S_2 & P_1A_d & 0 \\ * & \breve{A}^TP_2+P_2\breve{A}+L_1^TL_1+S_3 & 0 & P_2A_d \\ * & * & -S_1 & -S_2 \\ * & * & * & -S_3 \end{bmatrix}<0. \quad (19)$$

Using some changes of variables, $M_1=P_2\breve{A}$ and $M_2=P_2K$, the (19) is transformed into (i) of (11). Similarly to the procedure of proof (i) in (11), the proof of (ii) is completed using the Lyapunov functional (13) and continuous-time $H\infty$ performance measure (10). The linear matrix inequality (ii) in (11) implies that

$$\dot{V}(x_f(t))<-z_2(t)^Tz_2(t)+\gamma^2w(t)^Tw(t)<0. \quad (20)$$

Therefore we have

$$\begin{bmatrix} x_f(t) \\ x_f(t-d) \\ w(t) \end{bmatrix}^T \times \begin{bmatrix} A_f^TP+PA_f+C_{2f}^TC_{2f}+S & PA_{df} & PD_f \\ * & -S & 0 \\ * & * & -\gamma^2I \end{bmatrix}\begin{bmatrix} x_f(t) \\ x_f(t-d) \\ w(t) \end{bmatrix}<0. \quad (21)$$

Also, using the Schur complements and some changes of variables, the following matrix inequality

$$\begin{bmatrix} A_f^TP+PA_f+C_{2f}^TC_{2f}+S & PA_{df} & PD_f \\ * & -S & 0 \\ * & * & -\gamma^2I \end{bmatrix}<0 \quad (22)$$

is changed to (ii) in (11). Furthermore, by the integrating both sides of the inequality (15) from 0 to T_f and using the initial condition, we obtain

$$-\int_0^{T_f} z_1(t)^Tz_1(t)dt>x_f(T_f)^TPx_f(T_f)-x_f(0)^TPx_f(0) + \int_{T_f-d}^{T_f}x_f(\tau)^TSx_f(\tau)d\tau-\int_{-d}^{0}x_f(\tau)^TSx_f(\tau)d\tau. \quad (23)$$

As the closed loop system is asymptotically stable, when $T_f\rightarrow\infty$ some terms go to zero. Hence we get

$$\int_0^{\infty} z_1(t)^Tz_1(t)dt<\phi_f(0)^TP\phi_f(0)+\int_{-d}^{0}\phi_f(\tau)^TS\phi_f(\tau)d\tau. \quad (24)$$

This is an upper bound of *H2* performance measure. The first term of right hand side in (24) is changed to $-\alpha+\phi_f(0)^TP\phi_f(0)<0$. This is equivalent to (iii) in (11). The second term of the right hand side in (24) has the following relations

$$\begin{aligned}\int_{-d}^{0}\phi_f(\tau)^TS\phi_f(\tau)d\tau&=\int_{-d}^{0}tr(\phi_f(\tau)^TS\phi_f(\tau))d\tau \\ &=tr(NN^TS)=tr(N^TSN)<tr(Q).\end{aligned} \quad (25)$$

Therefore, $-Q+N^TSN<0$ is equal to (iv) in (11). ■ Hence, we can get the optimal continuous-time mixed *H2/H∞* filter satisfying the filtering design objective (8). Also, all solutions including filter variables($\breve{A}=P_2^{-1}M_1$, $K=P_2^{-1}M_2$) and the upper bound of *H*2 performance measure ($J^*=\alpha+tr(Q)$) can be calculated simultaneously because the proposed sufficient conditions are LMIs in terms of all finding variables.

Theorem 2: (Discrete time case) For a given positive constant γ, if the following optimization problem

$\min\{\alpha+tr(Q)\}$ subject to

$$i)\ \begin{bmatrix} -P_1 & 0 & P_1A & 0 & P_1A_d & 0 \\ * & -P_2 & P_2A-M_2C-M_1 & M_1 & 0 & P_2A_d \\ * & * & -P_1+S_1 & S_2 & 0 & 0 \\ * & * & * & -P_2+S_3+L_1^TL_1 & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 \\ * & * & * & * & * & -S_3 \end{bmatrix}<0,$$

$$ii)\ \begin{bmatrix} -P_1 & 0 & P_1A & 0 & P_1A_d & 0 & P_1B \\ * & -P_2 & P_2A-M_2C-M_1 & M_1 & 0 & P_2A_d & P_2B-M_2D \\ * & * & -P_1+S_1 & S_2 & 0 & 0 & 0 \\ * & * & * & -P_2+S_3+L_2^TL_2 & 0 & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 & 0 \\ * & * & * & * & * & -S_3 & 0 \\ * & * & * & * & * & * & -\gamma^2I \end{bmatrix}<0,$$

$$iii)\ -\alpha+\phi_1(0)^TP_1\phi_1(0)+\phi_2(0)^TP_2\phi_2(0)<0,$$

$$iv)\ -Q+N_1^TS_1N_1+N_2^TS_2N_1+N_1^TS_2N_2+N_2^TS_3N_2<0 \quad (26)$$

has a solution positive definite matrices(or scalar) P_1, P_2, S_1, S_2, S_3, α, Q, and matrices M_1, M_2, then (2) is a discrete time mixed *H2/H∞*filter and $J^*=\alpha+tr(Q)$ is an upper bound of discrete time *H2* performance measure. Here, some notations are defined as

$$\begin{aligned} M_1&=P_2\breve{A} \\ M_2&=P_2K \\ \sum_{i=-d}^{-1}\phi_f(i)\phi_f(i)^T&=NN^T=\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}[N_1^T\ \ N_2^T]. \end{aligned} \quad (27)$$

Proof: Similarly to the continuous time case, define a Lyapunov functional as follows:

$$V(x_f(t))=x_f(t)^TPx_f(t)+\sum_{i=t-d}^{t-1}x_f(i)^TSx_f(i). \quad (28)$$

The difference of the (28) is given

$$\begin{aligned}\Delta V&=V(x_f(t+1))-V(x_f(t)) \\ &=x_f(t+1)^TPx_f(t+1)-x_f(t)^TPx_f(t) \\ &\quad +x_f(t)^TSx_f(t)-x_f(t-d)^TSx_f(t-d).\end{aligned} \quad (29)$$

The linear matrix inequality (i) in (26) implies that

$$\Delta V<-z_1(t)^Tz_1(t)<0. \quad (30)$$

Therefore we have

$$\begin{bmatrix} x_f(t) \\ x_f(t-d) \end{bmatrix}^T \begin{bmatrix} A_f^T P A_f - P + S + C_{1f}^T C_{1f} & A_f^T P A_{df} \\ * & -S + A_{df}^T P A_{df} \end{bmatrix} \times \begin{bmatrix} x_f(t) \\ x_f(t-d) \end{bmatrix} < 0, \tag{31}$$

when assuming the zero noise signal vector input. And if we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix}, \tag{32}$$

then the following inequality

$$\begin{bmatrix} A_f^T P A_f - P + S + C_{1f}^T C_{1f} & A_f^T P A_{df} \\ * & -S + A_{df}^T P A_{df} \end{bmatrix} < 0 \tag{33}$$

is changed to

$$\begin{bmatrix} -P_1 & 0 & P_1 A & 0 & P_1 A_d & 0 \\ * & -P_2 & P_2 A - P_2 K C - P_2 \tilde{A} & P_2 \tilde{A} & 0 & P_2 A_d \\ * & * & -P_1 + S_1 & S_2 & 0 & 0 \\ * & * & * & -P_2 + S_3 + L_1^T L_1 & 0 & 0 \\ * & * & * & * & -S_1 & -S_2 \\ * & * & * & * & * & -S_3 \end{bmatrix} < 0. \tag{34}$$

Using some changes of variables, $M_1 = P_2 \tilde{A}$, $M_2 = P_2 K$, (34) is transformed into (i) of (26). Similarly to the procedure of proof (i) in (26), the proof of (ii) is completed using the Lyapunov functional (28) and discrete-time $H\infty$ performance measure (10). The linear matrix inequality (ii) in (26) implies that

$$\Delta V < -z_2(t)^T z_2(t) + \gamma^2 w(t)^T w(t) < 0. \tag{35}$$

Therefore we have

$$\begin{bmatrix} x_f(t) \\ x_f(t-d) \\ w(t) \end{bmatrix}^T \begin{bmatrix} A_f^T P A_f - P + S + C_{2f}^T C_{2f} & A_f^T P A_{df} & A_f^T P D_f \\ * & -S + A_{df}^T P A_{df} & A_{df}^T P D_f \\ * & * & -\gamma^2 I + D_f^T P D_f \end{bmatrix} \begin{bmatrix} x_f(t) \\ x_f(t-d) \\ w(t) \end{bmatrix} < 0. \tag{36}$$

Also, using the Schur complements and some changes of variables, the following matrix inequality

$$\begin{bmatrix} A_f^T P A_f - P + S + C_{2f}^T C_{2f} & A_f^T P A_{df} & A_f^T P D_f \\ * & -S + A_{df}^T P A_{df} & A_{df}^T P D_f \\ * & * & -\gamma^2 I + D_f^T P D_f \end{bmatrix} < 0 \tag{37}$$

is changed to (ii) in (26). Furthermore, by the summation both sides of the inequality (30) from 0 to $T_f - 1$, we obtain

$$-\sum_{t=0}^{T_f-1} z_1(t)^T z_1(t) > x_f(T_f)^T P x_f(T_f) - x_f(0)^T P x_f(0) + \sum_{i=T_f-d}^{T_f-1} x_f(i)^T S x_f(i) - \sum_{i=-d}^{-1} x_f(i)^T S x_f(i). \tag{38}$$

As the closed loop system is asymptotically stable, when $T_f \to \infty$ (or $T_f - 1 \to \infty$) some terms go to zero. Hence we get

$$\sum_{t=0}^{\infty} z_1(t)^T z_1(t) \le \phi_f(0)^T P \phi_f(0) + \sum_{i=-d}^{-1} \phi_f(i)^T S \phi_f(i). \tag{39}$$

This is an upper bound of $H2$ performance measure. The first term of right hand side in (39) is changed to $-\gamma + \phi_f(0)^T P \phi_f(0) < 0$. This is equivalent to (iii) in (26). The second term of right hand side in (39) has the following relations

$$\begin{aligned} \sum_{i=-d}^{-1} \phi_f(i)^T S \phi_f(i) &= \sum_{i=-d}^{-1} tr(\phi_f(i)^T S \phi_f(i)) \\ &= tr(NN^T S) = tr(N^T S N) < tr(Q). \end{aligned} \tag{40}$$

Therefore, $-Q + N^T S N < 0$ is equal to (iv) in (26). ■

Similarly to the continuous time case, we can get the optimal discrete time mixed $H2/H\infty$ filter satisfying the filtering design objective (8). Also, all solutions including filter variables($\tilde{A} = P_2^{-1} M_1$, $K = P_2^{-1} M_2$) and the upper bound of $H2$ performance measure($J^* = \gamma + tr(Q)$) can be calculated simultaneously because the proposed sufficient conditions are LMIs regarding all finding variables.

Remark: The proposed filtering design algorithms can be extended into the various kinds of continuous and discrete time systems such as multiple time delay systems, parameter uncertain time delay systems, convex bounded uncertain systems, and so on. Also, our filtering design methods include the guaranteed cost filtering design problems. Moreover, the presented continuous and discrete mixed $H2/H\infty$ filtering design algorithms can be applied to the system without time delay.

Example: Consider the following linear time delay system given by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.1 & 0 \\ 1 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} -0.01 & 0 \\ 0.1 & -0.05 \end{bmatrix} x(t-d) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \\ y(t) &= [1 \;\; 0]\, x(t) + w(t) \\ z_1(t) &= [1 \;\; 1]\, e(t) \\ z_2(t) &= [1 \;\; 2]\, e(t) \\ d &= 2, \quad \gamma = 1, \quad \phi_f(t) = [e^{t+1} \;\; 0 \;\; 0.1 \;\; 0]^T. \end{aligned} \tag{41}$$

All solutions in Theorem 1 are obtained using the LMI Toolbox[13] as follows:

$$P_1 = 10^{-4} \times \begin{bmatrix} 0.1806 & -0.0283 \\ -0.0283 & 0.0312 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 3.3239 & 3.6345 \\ 3.6345 & 4.6593 \end{bmatrix},$$

$$S_1 = 10^{-5} \times \begin{bmatrix} 0.4528 & -0.2355 \\ -0.2355 & 0.1612 \end{bmatrix},$$

$$S_2 = 10^{-4} \times \begin{bmatrix} 0.1346 & -0.1313 \\ -0.1313 & 0.6066 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 0.3673 & -0.2061 \\ -0.2061 & 0.1519 \end{bmatrix}, \quad \alpha = 0.0334,$$
$$Q = \begin{bmatrix} 0.0000 & 0 & 0 & 0 \\ 0 & 0.0056 & 0.0031 & 0 \\ 0 & 0.0031 & 0.0018 & 0 \\ 0 & 0 & 0 & 0.0000 \end{bmatrix}, \qquad (42)$$
$$M_1 = \begin{bmatrix} -1.3328 & -1.8173 \\ -0.3620 & -2.3296 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 4.6349 \\ 4.6578 \end{bmatrix}.$$

Therefore the stable continuous time filter and the upper bound of *H2* performance measure are

$$\dot{\hat{x}}(t) = \begin{bmatrix} -2.1490 & -0.0001 \\ 1.5987 & -0.4999 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -0.01 & 0 \\ 0.1 & -0.05 \end{bmatrix} \hat{x}(t-d) + \begin{bmatrix} 2.0491 \\ -0.5987 \end{bmatrix} y(t), \qquad (43)$$
$$J^* = 0.0408.$$

The obtained stable continuous mixed $H2/H\infty$ filter guarantees not only the minimization of $H2$ performance measure but also the $H\infty$ norm bound of the closed loop system within $\gamma(=1)$. In the case of discrete-time case, all solutions in Theorem 2 are given as follows:

$$P_1 = 10^{-5} \times \begin{bmatrix} 0.4409 & -0.0967 \\ -0.0967 & 0.2501 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.3303 & 1.7085 \\ 1.7085 & 8.9450 \end{bmatrix},$$
$$S_1 = 10^{-5} \times \begin{bmatrix} 0.0878 & 0.0141 \\ 0.0141 & 0.1368 \end{bmatrix},$$
$$S_2 = 10^{-4} \times \begin{bmatrix} 0.0292 & 0.0135 \\ 0.0135 & 0.3998 \end{bmatrix},$$
$$S_3 = \begin{bmatrix} 0.1741 & -0.0885 \\ -0.0885 & 0.0496 \end{bmatrix}, \quad \alpha = 0.0133,$$
$$Q = \begin{bmatrix} 0.0000 & 0 & 0 & 0 \\ 0 & 0.0027 & 0.0015 & 0 \\ 0 & 0.0015 & 0.0008 & 0 \\ 0 & 0 & 0 & 0.0000 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} -0.0108 & -0.8543 \\ -0.0640 & -4.4725 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1.5863 \\ 8.8381 \end{bmatrix}. \qquad (44)$$

Therefore the stable discrete time filter and the upper bound of $H2$ performance measure are

$$\hat{x}(t+1) = \begin{bmatrix} 0.0014 & 0.0000 \\ -0.0074 & -0.5000 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -0.01 & 0 \\ 0.1 & -0.05 \end{bmatrix} \hat{x}(t-d) + \begin{bmatrix} -0.1014 \\ 1.0074 \end{bmatrix} y(t), \qquad (45)$$
$$J^* = 0.0168.$$

The obtained stable discrete mixed $H2/H\infty$ filter guarantees minimization the upper bound of $H2$ performance measure and $H\infty$ norm bound within prescribed γ.

III. Conclusion

In this paper, we proposed the mixed $H2/H\infty$ filtering design algorithms for time delay systems in continuous time case and discrete time case, respectively. The sufficient conditions of the existence of filter and mixed $H2/H\infty$ filtering design method were presented using LMI approach. The proposed stable mixed $H2/H\infty$ filter guaranteed minimization the upper bound of $H2$ performance measure satisfying the $H\infty$ norm bound within γ. Also, we checked the validity of the proposed method by an example.

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