Control of Two-Link Manipulator Via Feedback Linearization and Constrained Model Based Predictive Control

Won-Kee Son, Jin-Young Choi, Hee-Seob Ryu, and Oh-Kyu Kwon

Abstract: This paper combines the constrained model predictive control with the feedback linearization to solve a nonlinear system control problem with input constraints. The combined approach consists of two steps: Firstly, the nonlinear model is linearized by the feedback linearization. Secondly, based on the linearized model, the constrained model predictive controller is designed taking input constraints into consideration. The proposed controller is applied to two link robot system, and tracking performances of the controller are investigated via some simulations, where the comparisons are done for the cases of unconstrained, constrained input in feedback linearization.

Keywords: Constrained model based predictive control, feedback linearization, two link robot manipulator, constraints handling

I. Introduction

In many common control problems involving nonlinear systems, it is possible to construct a feedback control law which will transform the original nonlinear plant into a closed loop system which is exactly linear. Once a nonlinear plant is linearized by feedback, all of the tools developed for the theory of linear systems can be directly applied to control the new closed loop. This method allows to design an exact linear controller for the nonlinear systems, which is affine in the control. However, if there is any constraint on input or output in the nonlinear system under consideration, it will not work owing to the constraint[1][2]. Another method for controlling nonlinear systems is the gain scheduling which is a technique that can extend the validity of the linearization approach to a range of operating points. But this method has a basic limitation that the controller is guaranteed to work only in the neighborhood of an equilibrium point.

One goal of the current paper is to design a control system for nonlinear systems with input constraints. A combined approach which combines the FL (Feedback Linearization) and the constrained model predictive control to control nonlinear dynamic system with constraints is proposed in this paper. In all physical systems there are limitations to the changes that can be made to the manipulated variables. Therefore, a control algorithm should be designed to have the ability to account for such limitations. To fulfill overall operational requirements, it is necessary to explicitly consider all constraints in the formulation of the control cost function. Accounting for such constraint in this paper, the CMBPC (Constrained Model Based Predictive Control)[3], is adopted as a control technique.

Another goal of this paper is to apply the combined approach to two-link robot system proposed by Spong and Vidyasagar[4]. The model is nonlinear with two inputs and two outputs. In the simulation, it is shown that the combined approach works well even under some constraints on input variable in the two-link robot system.

The layout of the paper is as follows: In Section II, the feedback linearization for nonlinear systems and model predictive control for systems subject to input constraints are reviewed. Then, a combined approach method, which combines the feedback linearization and the constrained model predictive control is proposed. The combined approach is applied to two link robot system in the constrained input through some simulations in Section III to exemplify the tracking performance of the scheme. Finally, conclusions are summarized in Section IV.

II. Combined control via FL and CMBPC

The basic concept of the combined control method is to linearize the nonlinear system and to solve the model predictive control problem for the linearized model. In general, by applying FL to constrained nonlinear system, the original optimization problem for nonlinear system subject to linear constraints is transformed into an optimization problem for a linear system subject to nonlinear constraints. Due to these constraints, the linearity of the linearized system established by FL is not preserved. Therefore, in order to preserve the linearity established by FL, an MPC-based method is proposed to handle these constraints in the controller design procedure, and a constraint mapping method is also presented to deal with nonlinear state-dependent constraints to be transformed by FL. The configuration of combined approach in presence of constraints on input is shown in Fig.1. The internal loop is concerned only with linearizing the plant and tracking its own reference input v. The external loop produces a control output v as a function of the external reference r and of the state z of the linearized model.

Firstly, the feedback linearization method for nonlinear systems is reviewed. Consider the nonlinear system having the number of m of inputs represented by the state equation

The closed loop system for control law with feedback linearization is represented in the block diagram in Fig.2.

Fig. 1. Configuration of combined control method in presence of constraint.

Fig. 2. Feedback linearization.

The combined control method involves discretization of the feedback linearized model for model predictive control design. The input and state transformation is performed at each sampling time using the discrete controller and current sampled states. It is well known that discretization represents an obstruction to exact feedback linearization[5]. The exact feedback linearization is not actually achieved because the controller is discretized. However, because the discretized control law uses approximated linear model in practice, the effect of discretization is neglected in this paper. The discretized linear model of feedback linearized system is given as follows:

$$
\begin{aligned}
z(k+1) &= A_d z(k) + B_d v(k) \\
y(k) &= C_d z(k),
\end{aligned} \tag{1}
$$

where the vector z denotes the system states, v the control input signals, and y the controlled signals. The matrices A_d, B_d, C_d contain the model parameters.

The control law for model-based predictive control should minimize the following cost function:

$$
J = \sum_{j=N_1}^{N_2} [r(k+j) - \hat{y}(k+j)]^2 + \lambda \sum_{j=0}^{N_u-1} [v(k+j)]^2. \tag{2}
$$

In the cost function (5), N_1 denotes the lower cost horizon, N_2 the upper cost horizon, N_u the control horizon, λ the weighting factor for control increments, and r the reference, $\hat{y}(k+j)$ the j-step ahead predictor. The well-known LS(Least Squares) solution solves this minimization problem and just the first element of the future control is a actually needed for the control law with the controller gain vector.

The minimization problem of the cost function with constraints is reformulated as solving QP(Quadratic Programming) problem. The algorithm uses SVD(Singular Value Decomposition) to firstly determine the unconstrained minimum of the cost function. If the specified constraints are violated, then LDP(Least Distance Programming) incorporating NNLS(Non-Negative Least Squares) is applied to find an admissible solution. Inspection of the cost function shows that the optimization problem being considered is Least Squares with linear Inequality constraints[3]:

$$
Min \parallel \Gamma v - b \parallel^2 \quad subject\ to\ \Im v \geq h, \tag{3}
$$

with

$$
\Gamma = \begin{bmatrix} H \\ \lambda^{\frac{1}{2}} I \end{bmatrix}, \quad b = \begin{bmatrix} r - f \\ 0 \end{bmatrix}.
$$

The free response f is defined as the predicted output when the control input remains constant from now on, and H is as follows:

$$
H = \begin{bmatrix} h_{N_1,1} & \dots & h_{N_1,N_u} \\ \vdots & \ddots & \vdots \\ h_{N_2,1} & \dots & h_{N_2,N_u} \end{bmatrix},
$$

$$
h_{j,i} = \begin{cases} C_d A_d^{j-i} B_d, & j \geq i \\ 0, & j < i. \end{cases}
$$

The matrix $\Im$ contains dynamic information about the constraints while h gives the limiting values for the constraints.

1. Constraints handling

Consider the nonlinear systems described in (1) with constraints on input

$$
u_{lc,i} \leq u_i \leq u_{uc,i} \quad i = 1, \dots, m, \tag{4}
$$

where $u_{lc,i}, u_{uc,i}$ are lower and upper bound, respectively. Then, we can define a new control variable v as an external reference input. Introducing a coordinate transformation and discretizing the continuous model (3), we can define the discrete linearized system as (4). Note that (4) hold the linear properties only if the constraints conditions on its input for $0 \leq j \leq N_u - 1$ are satisfied

$$
v_{lb}(k+j|k) \leq v(k+j|k) \leq v_{ub}(k+j|k), \tag{5}
$$

where $v(k+j|k)$ is the value of the input $v(k+j)$ computed at time step k, $v_{lb}(k+j|k)$ and $v_{ub}(k+j|k)$ are the constraints $v_{lb}(k+j)$ and $v_{ub}(k+j)$ computed at time step k, respectively. It is clear that, due to the feedback linearization, original hard constraints $(u_{lc,i}, u_{uc,i})$ are mapped into the MPC constraints on v that are in general *nonlinear and state dependent*. To avoid reaching these input constraints and to improve control performance, the MPC can be applied to the discrete linearized system given by (4). The mapping must be performed at each sampling instant because the mapping is state dependent. Moreover, the mapping must be accomplished over the entire control horizon of the liner MPC controller. The input constraint mapping is performed using the feedback linearization control law (2) and the current state measurement $x(k)$. This mapping can be written as follows:

$$
v(k) = \bar{\alpha}[x(k)] + \bar{\beta}[x(k)]u(k), \tag{6}
$$

where

$$
\bar{\alpha}[x(k|k)] = -\beta^{-1}(x)\alpha(x), \quad \bar{\beta}[x(k|k)] = \beta^{-1}(x).
$$

In the ideal case, for $0 \leq j \leq N_u - 1$ the transformed constraints at time step k are determined by solving the following optimization problem

$$
\begin{aligned}
v_{lb}(k+j|k) &= \min_{u(k+j|k)} v(k+j|k) \\
&= \min_{u(k+j|k)} \bar{\alpha}[x(k+j|k)] + \bar{\beta}[x(k+j|k)]u(k+j|k), \\
v_{ub}(k+j|k) &= \max_{u(k+j|k)} v(k+j|k) \\
&= \max_{u(k+j|k)} \bar{\alpha}[x(k+j|k)] + \bar{\beta}[x(k+j|k)]u(k+j|k), \tag{7}
\end{aligned}
$$

subject to the constraints

$$
u_{lb} \leq u(k+j|k) \leq u_{ub}. \tag{8}
$$

In (10) and (11), $u(k+j|k)$ and $x(k+j|k)$ represent the input $u(k+j)$ and state $x(k+j)$ computed at time step k, respectively. (10) is linear programing problem at each step k. Since inputs are coupled in MIMO systems, it is not easy to find a solution of (10) and the linear programing requires more computational burden. To find a solution more easily, the decoupling method is presented here. If the matrix $\bar{\beta}(x)$ at k-step has distinct real eigenvalues, it can be always transformed

into diagonal matrix $\bar{\beta}_d(x)$. If a matrix $\bar{\beta}(x)$ has repeated real eigenvalues at k-step, it is not always possible to find a diagonal matrix representation. Therefore, in this case, to diagonalize matrix $\bar{\beta}(x)$ at k-step, the following assumption is required.

Assumption 1: The $\mathcal{N}(\bar{\beta}_k - \lambda I)$ is equal to q, where q is the multiplicity of repeated real eigenvalue λ, $\mathcal{N}(\bar{\beta}_k - \lambda I)$ is the nullity of $(\bar{\beta}_k - \lambda I)$ and $\bar{\beta}_k$ is the constant valued matrix of $\bar{\beta}(x)$ at k-step.

Then, for $0 \leq j \leq N_u - 1$ the optimization problem in (10) can be reformulated as follows:

$$
\begin{aligned}
v_{lb}(k+j|k) &\Leftarrow \min_{u(k+j|k)} \bar{\beta}_d[x(k+j|k)]u(k+j|k), \\
v_{ub}(k+j|k) &\Leftarrow \max_{u(k+j|k)} \bar{\beta}_d[x(k+j|k)]u(k+j|k), \quad (9)
\end{aligned}
$$

subject to the constraints $u_{lb} \leq u(k+j|k) \leq u_{ub}$. Because exact mapping of future input constraints is impractical, it is necessary to approximate the constraints $v_{lb}(k+j|k)$ and $v_{ub}(k+j|k)$ for $j \geq 1$. Two approximate mapping techniques are discussed below.

1.1. Constant constraint technique

1) The most straightforward way to handle the constraint mapping problem is to simply extend the first input constraint over the entire control horizon.

$$
\begin{aligned}
v_{lb}(k+j|k) &= v_{lb}(k|k), \\
v_{ub}(k+j|k) &= v_{ub}(k|k), \quad 0 \leq j \leq N_u - 1. \quad (10)
\end{aligned}
$$

This mapping denotes tech1.

2) To assume all future control moves to be free of constraints, *i.e.*,

$$
\begin{aligned}
v_{lb}(k+j|k) &= \begin{cases} v_{lb}(k|k), & j = 0; \\ -\infty, & \text{otherwise} \end{cases} \\
v_{ub}(k+j|k) &= \begin{cases} v_{ub}(k|k), & j = 0; \\ \infty, & \text{otherwise} . \end{cases}
\end{aligned} \quad (11)
$$

This mapping denotes tech2. 1.2. Variable constraint technique   The predicted values of the transformed state variable is obtained as follows:

$$
\begin{aligned}
\hat{z}(k+j|k) &= A_d\hat{z}(k+j-1|k) + B_dv(k+j-1|k-1) \\
\hat{z}(k|k) &= z(k). \quad (12)
\end{aligned}
$$

The state sequences and the inverse transformation $T_s^{-1}(z)$ are used to compute future values of the actual state vector

$$
\begin{bmatrix} \hat{x}(k|k) \\ \hat{x}(k+1|k) \\ \vdots \\ \hat{x}(k+N_u-1|k) \end{bmatrix} = \begin{bmatrix} x(k) \\ T_s^{-1}[\hat{z}(k+1|k)] \\ \vdots \\ T_s^{-1}[\hat{z}(k+N_u-1|k)] \end{bmatrix} . \quad (13)
$$

The optimization problem in (10) is solved by substituting the predicted state variables $x(k+j|k-1)$ in place of $x(k+j|k)$. The solution yields the transformed constraints. These variable constraints are used in the linear MPC design in place of the constraints h in (6). The procedure is repeated at the next time step with the input sequences and the measurement $x(k+1)$. If the linear MPC is infeasible, constraints are dropped on the final input in the control horizon, $v(k+N_u-1|k)$, and the problem is resolved. If the problem remains infeasible, constraints are

Fig. 3.   Block diagram of combined scheme with variable constraint technique.

Fig. 4. Two link robot manipulator.

dropped on the last two inputs, $v(k+N_u-2|k)$ and $v(k+N_u-1|k)$. The process is continued until feasibility is achieved.

III. Design example

The proposed controller is applied to the two link robot system proposed by Spong and Vidyasagar[4]. The robot system is described by 2×2 nonlinear model.

With $\mathbf{q} = [q_1 \; q_2]^T$ being the two joint angles, $\tau = [\tau_1 \; \tau_2]^T$ being the joint inputs, two link robot manipulator can be generally expressed as

$$
\mathbf{H(q)\ddot{q}} + \mathbf{C(q,\dot{q})\dot{q}} + \mathbf{g(q)} = \tau,
$$

where $\mathbf{H(q)}$ is the 2×2 manipulator inertia matrix (which is symmetric positive definite), $\mathbf{C(q,\dot{q})\dot{q}}$ is a 2-vector of centripetal and Coriolis torques (with $\mathbf{C(q,\dot{q})}$ a 2×2 matrix), $\mathbf{g(q)}$ is the 2-vector of gravitational torques, and τ is the actuator input torques applied to the manipulator joints. Let us choose $[q_1 \; \dot{q}_1 \; q_2 \; \dot{q}_2]^T$ as states $[x_1 \; x_2 \; x_3 \; x_4]^T$. Then, nonlinear system is expressed as follows:

$$
\begin{aligned}
&\begin{bmatrix} H_{11} & H_{12}\cos(x_1-x_3) \\ H_{21}\cos(x_1-x_3) & H_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} \\
&+ \begin{bmatrix} 0 & h\sin(x_1-x_3)x_4 \\ -h\sin(x_1-x_3)x_2 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} \\
&+ \begin{bmatrix} g_{11}\sin(x_1) \\ g_{22}\sin(x_3) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad (14)
\end{aligned}
$$

where

$$
\begin{aligned}
H_{11} &= m_1l_{c_1}^2 + m_2l_1^2 + I_1, \quad H_{22} = m_2l_{c_2}^2 + I_2 \\
H_{12} &= H_{21} = m_2l_1l_{c_2}, \quad h = m_2l_1l_{c_2} \\
g_{11} &= -[m_1l_{c_1} + m_2l_1]g, \quad g_{22} = -m_2l_{c_2}g. \quad (15)
\end{aligned}
$$

The state transformation $z = T_s(x)$ is given by

$$
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = T_s(x) = \begin{bmatrix} x_1 \\ x_3 \\ x_2 \\ x_4 \end{bmatrix}. \quad (16)
$$

And inverse transformation T_s^{-1} also is given by

$$
x_1 = z_1, \quad x_2 = z_3, \quad x_3 = z_2, \quad x_4 = z_4. \quad (17)
$$

Table 1. Set parameters for two link manipulator.

Parameter	Value	Parameter	Value
$m_1[kg]$	10	$m_2[kg]$	5
$l_1[m]$	1	$l_2[m]$	1
$l_{c_1}[m]$	0.5	l_{c_2}	0.5
$I_1[Nms^2/rad]$	0.833	$I_2[Nms^2/rad]$	0.417
$g[m/s^2]$	9.8		

(d)
The
case
in
which
in-
puts
are
cou-
pled-
pled

Fig. 5. The decoupling of coupled inputs.

(a)
The
an-
gle
of
link1

(b)
The
an-
gle
of
link2

(c)
The
torque
of
in-
put1

(d)
The
torque
of
in-
put2

Fig. 6. Tracking performance for square reference.

The input transformation is achieved as follows :

$$\left[\begin{array}{c} \tau_1 \\ \tau_2 \end{array}\right] = \beta(x)v + \alpha(x)$$

$$= \left[\begin{array}{cc} H_{11} & H_{12}\cos(x_1 - x_3) \\ H_{21}\cos(x_1 - x_3) & H_{22} \end{array}\right]\left[\begin{array}{c} v_1 \\ v_2 \end{array}\right]$$

$$+ \left[\begin{array}{cc} 0 & h\sin(x_1 - x_3)x_4 \\ -h\sin(x_1 - x_3)x_2 & 0 \end{array}\right]\left[\begin{array}{c} x_2 \\ x_4 \end{array}\right]$$

$$+ \left[\begin{array}{c} g_{11}\sin(x_1) \\ g_{22}\sin(x_3) \end{array}\right]. \tag{18}$$

As a result of the above state and input transformations, we obtain the following set of linear equations $\dot{z} = Az + Bv$

$$\left[\begin{array}{c} \dot{z}_1 \\ \dot{z}_2 \end{array}\right] = \left[\begin{array}{c} z_3 \\ z_4 \end{array}\right], \quad \left[\begin{array}{c} \dot{z}_3 \\ \dot{z}_4 \end{array}\right] = \left[\begin{array}{c} v_1 \\ v_2 \end{array}\right],$$

where

$$A = \left[\begin{array}{cc} 0 & I \\ 0 & 0 \end{array}\right], \quad B = \left[\begin{array}{c} 0 \\ I \end{array}\right]. \tag{19}$$

The feedback linearized system (22) is discretized with sampling time T to facilitate the subsequent MPC design, then the discrete feedback linearized system has the form in (4).

1. Simulation results

In two link robot manipulator, we use the coefficients in Table.1. The linearized model (22) is discretized with sampling time $0.01[sec]$ to use the model predictive controller, and the zero order hold is used to hold. And it is assumed that the full states are measurable and that the input torques of two link manipulator robot system are limited to $-200[Nm] \leq u_1 \leq 200[Nm]$ and $-100[Nm] \leq u_2 \leq 100[Nm]$. When the constrained actual inputs u are coupled and decoupled, the maximum and minimum values of the linearized system inputs v to solve the optimization problem respectively in (10) and (12) is shown in Fig.5. We see from this figure that if the $\bar{\beta}(x)$ is decoupled the u_1, u_2 are orthogonal. Therefore, we can obtain the solution easily by solving (12) instead of the original optimization problem (10). Then the lower and upper bounds, $v_{lb}(k+j|k), v_{ub}(k+j|k)$, for the new input v of the linearized system can be calculated by (12). The model predictive controller tuning parameters are chosen with a lower cost horizon $N_1 = 2$, a upper cost horizon $N_2 = 20$ and a control horizon $N_u = 2$, by trial and errors.

(a)
The
an-
gle
of
link1

(b)
The
an-
gle
of
link2

(c)
The
torque
of
in-
put1

(d)
The
torque
of
in-
put2

Fig. 7. Tracking performance for Sinusoidal reference.

Simulation results for square reference are shown in Fig.6 and for sinusoidal reference are also shown in Fig.7. We can see that when the controller uses input constraint that are constant over the control horizon, it yields a poor reference tracking as the outputs oscillate and overshoot the reference. In this method(tech1, tech2), the actual input $u(k)$ mapped by the first pair of constraints, $v_{lb}(k)$ and $v_{ub}(k)$, satisfy the actual constraints (7). However, the constraints $v_{lb}(k+j)$ and $v_{ub}(k+j)$ may not lead to inputs $v(k+j)$ that satisfy the actual input constraints (7). Therefore, This mapping methods have a disadvantage that incorrect future constraints may map to implemented control that are unnecessarily aggressive. Due to this disadvantage, we can see the spike in actual input torques of Fig.7. By contrast, the controller which employs constraint that vary over the control horizon yields a fast, improved tracking responses.

IV. Conclusion

In this paper a combined control scheme combining the constrained model predictive control with feedback linearization is presented, and its performance is analyzed via some simulation applied to two link robot system, which has shown that the controller presented works well even under some actuator saturation. The simulation also has compared the performance of combined scheme in the case of constrained input with that of the case with unconstrained input. However, the control method adopted in the current paper has not taken the parameter uncertainty into consideration, and so it requires further research to develop robust control methods.

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Won-Kee Son

He was born in Korea, in 1969. He received B.S., M.S., and Ph.D. degrees in Electrical Engineering from Inha University in 1995, 1997 and 2000, respectively. Since 2000, he has been Post-Doctoral in School of Electrical Engineering, Seoul National University. His main research interests are in the areas of predictive control, fault tolerant control, nonlinear control and digital control applications.

Jin-Young Choi

He received the B.S., M.S. and Ph.D. degrees in Control and Instrumentation Engineering from Seoul National University in 1982, 1984, 1993, respectively. From 1992 to 1994, he was with the Basic Research Department of ETRI, where he was a senior member of technical staff working on the neural information processing system. Since 1994, he has been with Seoul National University, where he is currently Assistant Professor in the School of Electrical Engineering.

Hee-Seob Ryu

He was born in Seoul, Korea, in 1971. He received B.S. and M.S. degree in Electrical Engineering from Inha University in 1994 and 1996, respectively. He is currently a Ph. D. candidate in the School of Electrical Engineering, Inha University. His main research interests are in the areas of robust estimation, control, and industrial applications.

Oh-Kyu Kwon

He was born in Seoul, Korea, in 1952. He received B.S. and M.S. degree in Electrical Engineering and his Ph.D. degree in Control and Instrumentation Engineering from Seoul National University in 1978, 1980 and 1985, respectively. His main research interests are in the areas of robust estimation, fault detection and diagnosis, system identification and digital control applications.